Abstract

Given a noisy and incomplete point set, we introduce a method that simultaneously recovers a set of locally fitted primitives along with their global mutual relations. We operate under the assumption that the data corresponds to a man-made engineering object consisting of basic primitives, possibly repeated and globally aligned under common relations. We introduce an algorithm to directly couple the local and global aspects of the problem. The local fit of the model is determined by how well the inferred model agrees to the observed data, while the global relations are iteratively learned and enforced through a constrained optimization. Starting with a set of initial RANSAC based locally fitted primitives, relations across the primitives such as orientation, placement, and equality are progressively learned and conform to local data. The global coupling corrects the primitives obtained in the local RANSAC stage, and brings them to precise global alignment. We test the robustness of our algorithm on a range of synthesized and scanned data, with varying amounts of noise, outliers, and non-uniform sampling, and validate the results against ground truth, where available.

Keywords: 3D scanning, RANSAC, global relations, data fitting, symmetry relations.

Figure 1: Starting from a noisy scan, our algorithm recovers the primitive faces along with their global mutual relations, when are then used to produce a final model (all lengths in mm).

Figure 2: Man-made objects commonly consist of primitive faces conformation to various global relations.

A popular strategy in reverse engineering involves locally fitting primitives like planes, cylinders, cones using state-of-the-art RANSAC based methods [Schnabel et al. 2009]. Such a local approach, by itself, can be unreliable, especially in regions of biased noise or incomplete data leading to globally inconsistent reconstructions, and hence poor proxies for the corresponding mechanical parts. We argue that unlike local relations, global ones are less easily disturbed. Further, such relations being non-local span a wider extent of the object, and thus are more robust to local inconsistencies. In this paper, we present a framework to learn and conform to global relations (see Figures 1 and 4).

Existing approaches typically make use of smoothness priors to process incomplete and noisy data. Alternately, Gal et al. [2007] use local priors to fit primitive shapes like boxes, cylinders, cones to the scanned data. While the strategy can produce sharp features using those inherited from the primitive shapes, the method being local fails to conform to global relations, which constitute essential characteristics of mechanical parts. Further, such approaches typically necessitate committing to a partitioning of the input early on, posing an additional challenge.

In this paper, we use global relations for recovering exact relations and constraints from imperfect acquisitions of man-made engineer-
Our algorithm transforms the noisy input point data (left) to the final reconstructed model (right) through several iterations of RANSAC fitting and constrained optimization. The optimization achieves global alignment in three sequential stages, with each enforcing a different class of global relations (orientation, placement, or equality), while always accounting for the underlying data, and respecting relations conform to in the previous stages. The final result is visualized with the fitted planes clipped by neighboring ones.

Our algorithm directly couples the local and global aspects of the problem. The local fit of the model is determined by how well the inferred model agrees to the observed data, while the global relations are iteratively learned, and enforced via a constrained optimization. We favor alignment-relations involving orientation, placement, and equality across the primitives. The recovered global priors boost the local primitive fitting to a noise-resilient global primitive fitting (see Figure 12).

Primitives extracted by local RANSAC are unstable specially in regions of limited support. A naive sequential alignment to dominant primitives is insufficient as such an approach ignores the inter-relations across the relations themselves (see Figure 4). We take a global approach to constrain and optimize the local RANSAC based primitives. The problem is challenging since we simultaneously learn and infer valid relations among uncertain global constraints, while avoiding conflicts. We show that balancing between the data driven fitting error and the regularization effects of the inferred relationship structure, allows us to capture both local and global information directly from incomplete and noisy scans. In each stage, we extract a set of valid relations and use a nonlinear data error optimizer minimizing subject to the inferred constraints to simultaneously align all the primitives. Our algorithm works from coarse to fine scale, wherein relations from the coarse scale help position the unstable primitives extracted in the fine scale. We demonstrate the robustness of our algorithm on a variety of synthetic and scanned models subject to a wide range of data contaminations.

2 Related Work

Surface reconstruction from a scanned, unstructured point cloud is a difficult and ill-posed problem (see also [Dey 2007]). Usual strategies require high quality input data, restrict the problem space by focusing on specific object classes, or use prior knowledge about the scanned objects.

The pioneering work of Hoppe et al. [1992] computes a signed distance field to the underlying surface using local tangent plane construction, and extracts an isosurface from the distance field using the marching cubes algorithm. Over the years, other implicit formulations have been proposed, including radial basis function [Carr et al. 2001], Moving Least Squares (MLS) [Alexa et al. 2003], thin plate splines [Brown and Rusinkiewicz 2004], Poisson reconstruction [Kazhdan et al. 2006], and recently global stochastic signing of distance functions [Mullen et al. 2010]. These methods employ a smoothness prior to regularize the solution by favoring smooth reconstructions. Such an assumption is unsuited in many man-made and CAD objects, which often are piecewise smooth and contain characteristic sharp features.

Alternate strategies have been proposed to recover sharp features. Methods that rely on an explicit representation of sharp features [Guennebaud and Gross 2007] classify the input samples into piecewise linear components to indirectly model sharp features. A more challenging task is to detect and reconstruct the sharp features present in noisy point clouds [Fleishman et al. 2005; Lipman et al. 2007; Oztireli et al. 2009]. These methods are typically local and ignore global characteristics of the subject.

Man-made objects are often composed of an assembly of basic primitive shapes conformed using global relations such as regularity and intra-symmetry relations. This is particularly true for architectural, CAD and mechanical models, which are predominantly made of repetitive basic structures to facilitate easy and economic fabrication. Surface reconstruction involving local fitting of primitive structures has long been the standard in reverse engineering [Benko et al. 2001] and also in the graphics community (cf. [Schnabel et al. 2007] and references therein). These methods support reconstruction of shapes consisting of sharp features through local fitting of surface geometry. In contrast, we take a global approach which accounts for the inter-primitive relations, both local and global.

In computer vision and reverse engineering, Fisher and colleagues (see [Weghi et al. 2002; Fisher 2004] and references therein) demonstrate the effectiveness of domain knowledge of standard shapes and relationships for improved architectural reconstruction, better parameter estimation, and data completion using non-local relations (cf. [Thrun and Wegbreit 2005]). The series of work uses prior knowledge of specific buildings, placement constraints, or extracted features like principal directions for reconstructing buildings from range images. Guided by a similar philosophy, we also make use of global relations and placement constraints, which are automatically extracted by our algorithm.

Starting from an input scan, Gal et al. [2007] use multi-scale partial matching to fit a small set of basic shapes to local neighborhoods as local priors. To facilitate reconstruction, the scan is augmented with noise-free samples, quality normals, and sharp features from the matched primitive shapes. Schnabel et al. [2009] present an interesting hole-filling algorithm that is guided by primitive detection
in the input point cloud. Since direct detection of good geometric primitives is difficult due to noise and missing data, they minimize an energy function defined over a guiding vector field and employ a graph-cut based approach to partition the data. In contrast, we detect global relations in the input scan, encode them in a dynamically constructed relationship graph structure, and conform to the learned relations using a data driven (non-linear) constrained optimization.

Our work is inspired by other efforts that are not directly related to reconstruction: Mitra et al. [2007] use symmetrization to bring shapes into canonical poses in an effort to enhance the (approximate) Euclidean symmetries present in the inputs. However, reliable detection of global relations in presence of significant noise and outliers remain a challenging problem (cf. [Pauly et al. 2008; Li et al. 2010] and references therein). Gal et al. [2009] propose an intuitive manipulation framework for man-made objects by analyzing their typical features and characteristics. They use wires to encode local geometric characteristics as well as global relationships across the wires, which are greedily optimized during deformations. The wires, in turn, are augmented with geometric attributes allowing coupled manipulation of the input. Our goal is to extract structural relations from uncertain data, while [Gal et al. 2009; Mehra et al. 2009] focus on using such relations. Detected symmetries and relations have also been used for inverse procedural modeling [Mitra and Pauly 2008; Bokeloh et al. 2010]. In our work, we also incorporate local reconstruction coupled with an extracted relationship graph in order to encode the intra-part relations in the input man-made shape.

3 Algorithm

Given a noisy scanned point set, we simultaneously recover a set of fitted basic primitives along with their global mutual relations. The algorithm partitions the data, extracts the fitted primitives, and simultaneously learns and enforces the mutual relations, among both near and distant parts. While current approaches focus primarily on local primitive fitting, and ignore the global alignments due to the global relations, our method balances between the two.

We focus on three classes of commonly encountered relations, also referred to as alignments: (i) orientation relations like parallelism, orthogonality, regular angles, (ii) placement relations particularly coplanarity and coaxis relations between primitives, and (iii) equality relations among primitives. We solve a global problem to simultaneously learn which constraint relations to enforce without introducing conflicts, and optimize for the parameters of each primitive to meet the learned constraints, while conforming to the input (see Figure 3). The problem is challenging since we have to choose among uncertain relations. Based on the sensitivity of the various relations to noise, we detect them sequentially, starting with the most robust one. We perform global alignments one after another, without breaking already learned and satisfied relations.

State-of-the-art solutions, e.g., Schnabel et al. [2007], typically use a local RANSAC based approach to decompose the input points into subsets associated with fitted primitive shapes and a set of unclaimed points. To account for global relations, a naive option is to take the local primitive fits and progressively fix them, satisfying relations with the primitives having higher confidence. In each step of such an approach, a single primitive is positioned, and this placement is fixed during the subsequent steps. Further, primitives with lower confidence do not influence those with higher confidence. Such a greedy approach can be unreliable and fragile as one is forced to commit early to uncertain global relations (see Figures 4 and 12). Instead, we take a more global view leading to a robust primitive fitting, while simultaneously balancing among multiple constraints.

Let \( P := \{p_1, p_2, \ldots \} \) be the input point set, with each point \( p_i \) equipped with a normal \( n_i \) and a confidence score \( w_i \) estimated using a standard local covariance analysis. Note that we do not require consistently oriented normals. To bootstrap the algorithm, we partition the input into sets \( P = \bigcup P_i \), where each point group \( P_i \) has an associated primitive, say \( \chi_i \), along with initial estimates of its respective parameters, and \( P^* \) denotes the remaining points [Schnabel et al. 2007]. Later, we repartition the inputs based on the learned relations (see Figure 10).

For each of the alignment classes, we first construct a relationship graph. For example, in case of one type of orientation constraints, we check the angles between the normals of all pairs of primitives and this placement is fixed during the subsequent steps. Further, primitives with lower confidence do not influence those with higher confidence. Such a greedy approach can be unreliable and fragile as one is forced to commit early to uncertain global relations (see Figures 4 and 12). Instead, we take a more global view leading to a robust primitive fitting, while simultaneously balancing among multiple constraints.

Having extracted the non-redundant and non-conflicting relations \( C^* \), along with the relations learned in the previous stages, we solve a nonlinear optimization over the parameters of primitives \( \chi_i \) to minimize data error while exactly conforming to the constraint set \( C^* \). In this stage, all the parameters of the primitives are simultaneously optimized for, while keeping the point association to the respective sets \( P_i \) fixed.
In an outer loop, we identify the input points that are explained by
the current set of learned primitives, remove them from the input
set \( P \), and repeat the algorithm for the remaining unclaimed points.
Thus, we repartition the data based on the learned relations, and in
the process improve the data partitioning that is difficult to perform
just based on local reasoning. We stop once the remaining point set
size is small, the potential relationship graph is empty, or when we
reach a maximum number of iterations, 3 in our experiments (see
Figure 10).

In our implementation, we use planes, cylinders, spheres, and right
round cones as basic primitives. We parameterize each primitive
by an associated orientation direction and attribute(s) as follows: A
plane is represented in the normal-intercept format using a direction
\( \mathbf{n} \) and a signed distance from origin \( d \); a cylinder is represented
by its axis \( \mathbf{n} \), a point on the axis \( \mathbf{p} \), and a radius \( r \). A cone is represented
by its axis \( \mathbf{n} \), its apex \( \mathbf{p} \), and an apex angle \( \phi \); a sphere by its center
\( \mathbf{p} \) and radius \( r \). Each such primitive \( \mathcal{P} \) has an associated set of
points \( P \) that are used to clip the respective primitive. We clip
a plane using the convex hull of its projected member points, we clip
a cylinder or a cone by bounding the projected member points using
plane(s) aligned along its respective primitive axis, and a sphere by
a pair of parallel planes (see Figure 6). We next describe how we
detect and align to global orientation relations by optimizing over
the primitive parameters, while conforming to the input data.

### 3.1 Orientation alignment

**a) Parallel and orthogonal relations.** We now proceed as fol-
 lows: (i) identify candidate relations \( C_0 \) between the primitives
\( \mathcal{P} \), (ii) extract a maximal set of non-conflicting relations \( C_0 \subset C_0 \), and
(iii) use a constrained nonlinear optimization to align the primitives
to \( C_0 \) and the data.

**i) Candidate relations.** We create an undirected graph \( G_o := (V_0, E_0) \) with each primitive \( \mathcal{P} \) represented as a node \( v_i \in V_0 \). We
classify edge \( e_{ij} \in E_0 \) based on the angle \( \theta_{ij} = \cos^{-1}(\mathbf{n}_i \cdot \mathbf{n}_j) \)
between primitives \( \mathcal{P}_i \) and \( \mathcal{P}_j \). If \( \theta_{ij} \leq \pi/4 \), then \( e_{ij} \) is a parallel
relation-edge with a score \( s_{ij} = -\theta_i \); else, \( e_{ij} \) is an orthogonal
relation-edge with a score \( s_{ij} = (\theta_i - \pi/2) \). We leave out weak
edges based on a default threshold, \( \pi/12 \) for moderate noise level.
We thus have a set of candidate relations \( C_o := \{ e_1, e_2, \ldots \} \) sorted
according to their decreasing scores.

**ii) Extracting \( C_0 \).** A graph is said to be biconnected if remov-
ing a node, called cut-vertex, increases the number of connected
components. In the context of the graph \( G_o \), cut-vertices represent
the outlier primitives with chance alignment to isolated primitives,
since outliers in this context are primitives with comparatively few
relations. Unless removed, they can adversely influence subse-
quent optimization. We decompose \( G_o \) into bi-connected com-
ponents [Hopcroft and Tarjan 1973] and operate relations in each con-
ected component individually, henceforth referred to as \( C_o \).
Note that although there are no explicit relations across the components,
the primitives are held together by a data fitting term.

Any constraint set \( C_o \) may possibly contain conflicts making it in-
fefensible to optimize the primitives such that all the relations in \( C_o \)
are simultaneously fulfilled, i.e., the aligned primitives exactly satisf-
ies the relations. Since the input data is imprecise, accumulated
error can lead to erroneous relations or conflicts, e.g., in course of
reduction three nodes can be wrongly connected by an orthogonal
and two parallel relations (see Figure 7). One could use a weighted
least squares solution to approximately align to relations in \( C_o \), but
the result can easily be unsatisfactory since there might be outlier
constraints, none of the relations will be truly satisfied but only ap-
proximated (see Figure 12). Instead, we directly construct a subset
\( C_o \subset C_o \) without redundancy and free of conflicts, e.g., preventing
multi-edges and self-loops in the reduced relationship graph. Our
construction is based on the following properties of the maximal
subset \( C_o^* \):

**Figure 7:** In course of reduction of graph \( G_o \), conflicting edges
(A, B, C, A, B, C, A, B, C, A) can be created. In such a case, we remove the edge with lower
confidence, in this example BC (3), after parallel collapse B → A.
(i) No two nodes are connected by a parallel-relation edge, i.e., all edges are orthogonal-relation edges, otherwise we can collapse nodes joined by the parallel-relation edge, and thus remove redundancy.

(ii) A clique in the graph \( C_o \) can have a maximal size of three, since any orthogonal basis in 3D has only three elements.

(iii) If two graph loops share an edge, say ABC and ACD, then edge BD must necessarily be a parallel edge, otherwise they are in conflict. Note this requirement is a stronger condition compared to the non-existence of a 4-clique.

These properties motivate the following algorithm: Starting with \( C_o \leftarrow \emptyset \), we progressively add relations \( c_k \in C_o \) to \( C_o \) according to the decreasing scores of \( c_k \) based on reduction steps (see Figure 5):

- For any parallel edge \( c_k := e_{ij} \), we collapse node \( v_j \) to \( v_i \), and update the edge connectivity accordingly.

- If adding an orthogonal edge \( c_k \) results in 3-length loops ABC and ACD, then we check whether a parallel edge BD exists in the remaining \( C_o \) set. If parallel BD edge is present, we add \( c_k \) and BD to \( C_o \) and trigger a parallel edge collapse. Otherwise, we have a conflict and remove the lowest confidence edge involved.

- If the candidate edge \( c_k \) is in conflict with an existing edge, which is formed by result collapses in previous steps, then we ignore \( c_k \).

The construction stops once all the relations in \( C_o \) have been considered. Figure 5 shows a typical reduction sequence.

A) Aligning to \( C_o \). Given a point cluster \( P_i \) along with its associated primitive \( \chi_i \), we measure the data fitting error as,

\[
E_d(P_i, \chi_i) := \sum_{p \in P_i} w_p d^2(p, \chi_i),
\]

where \( d(p, \chi_i) \) measures the distance of point \( p \), with weight \( w_p \), to the primitive \( \chi_i \) [Kanatani 2008]. Then the accumulated data error is simply given by \( \sum_i E_d(P_i, \chi_i) \). Our goal is to minimize the data error, while aligning to the relations in \( C_o \). Since we have only orthogonality constraints at this stage, alignment to any such constraint \( e \in C_o \) between primitives \( \chi_i, \chi_j \) implies \( g_e := n_i^T n_j = 0 \). Additionally, for each primitive \( \chi_i \), the normal direction satisfies \( n_i^T n_i = 1 \). Thus we have the non-linear optimization with equality constraints,

\[
\min_{\{e\}} \sum_{i} E_d(P_i, \chi_i), \text{ such that, } g_e = 0 \forall e \in C_o, \ n_i^T n_j = 1 \forall j. \quad (1)
\]

We use a trust region method based on interior point nonlinear programming to solve the optimization [Byrd et al. 2000; Ziena 2010].

At the end of this stage, we have learned and aligned to orthogonal and parallel relations among the primitives.

b) Equal angle relations. Regular structures are common in man-made objects. Such regularity often results in equal angle constraints across primitive parts. Unlike parallel or orthogonal constraints, equal angles, however, are not known in advance, and are only recovered in course of optimization. A possible approach is to first cluster angles between primitive pairs, and then rank the primitive pairs based on the deviation of their mutual angle with the cluster peaks, similar to the parallel/orthogonal case. However, we found this approach to be unreliable due to presence of outliers, and spurious peaks in the angle space, e.g., histogram in Figure 8 shows why it is challenging to select clustering parameters in our setting. Instead, we directly detect and align to equal angle relations.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8}
\caption{Starting with primitives obtained using RANSAC, orientation alignment first conforms to parallel and orthogonal relations. In addition, in the next stage, equal angle relations are learned and aligned to. The histogram of angles between primitive pairs (range \([0 – \pi/2]\)) progressively gets cleaned along the stages of the algorithm.

without committing to a desired angle value, while still conforming to existing parallel/orthogonal relations. Any angle relation involves a pair of primitive pairs, i.e., four primitives. Thus, we build a graph \( G_c \), where each vertex represents an unordered pair of primitives. In noisy data, spatially distant primitives easily pollute the space with spurious relations. Hence, if primitives \( \{\chi_i, \chi_j\} \) are farther than 10% of the larger bounding box length, we remove its corresponding node. As a result in most cases graph \( G_c \) has only \( O(m) \) nodes, instead of \( O(m^2) \). For any pair of such vertices, say involving primitive pairs \( \{\chi_i, \chi_j\} \) and \( \{\chi_k, \chi_l\} \), we add an edge based on the similarity of their pairwise angles. Note that primitives \( \{\chi_i, \chi_j\} \) and \( \{\chi_k, \chi_l\} \) can be at arbitrary distances. The corresponding relation \( e \) is assigned a score \( s_e = -\langle n_i, n_j \rangle - \langle n_k, n_l \rangle \) with the associated constraint \( g_e := (n_i^T n_j)^2 - (n_k^T n_l)^2 = 0 \). We leave out edges if the subtended angle is more than a threshold, \( S \leq 10^\circ \) in our experiments. The threshold depends on the noise margin against the separation among the modes in the ground truth data, i.e., the regularity of the object. We collect the relations as \( C_e = \{e_1, e_2, \ldots\} \). Now starting with a set \( C_e \leftarrow \emptyset \), we progressively add edges \( e \in C_e \), based on decreasing confidence. Exact
equality relation being transitive, the relation graph should be free of cycles. Hence, we only add $c \rightarrow C_k$ if no cycles are formed. This step significantly decreases the number of graph edges, while retaining the confident ones. Set $C_k$ typically consists of $O(m)$ edges.

The set $C_k$ can still contain conflicts, and hence we use interior point nonlinear programming to detect such cases. We test if the optimization of the data energy (Equation 1) has any feasible solution subject to the constraints $C_k$. If no feasible solution exists, we take out the relation $c \in C_k$ with the lowest score, and test for feasibility again with the remaining set of relations. Typically 1-2 such relations were dropped in our experiments.

Figure 8 shows a typical scenario for orientation optimization. The initial graph $G_0$ has 84 edges, which gets reduced to 21 relations in $C_0$. In the equal angle stage, 33 initial candidate relations are recovered and conformed to while minimizing the data error. A feasible solution was found without discarding any relation of $C_k$. Total time taken was 680 seconds with most time spent in the constrained solver.

### 3.2 Placement alignment

In most man-made objects, coplanarity and coaxial relations carry important relation cues about the object parts (see also [Gal et al. 2009]). We conform to placement relations after orientation alignment, while preserving the orientation relations already aligned to.

Coaxial relations are simple to detect and enforce. Two primitives $g_i$ and $g_j$, e.g., two cylinders, or a cylinder and a cone, are potentially coaxially aligned if their axes are exactly parallel, since we already have orientation alignment. We assign a score to such a potential primitive pair based on the distance between their axes, namely, $s_{pq} = -\frac{1}{2} \cdot \left( \frac{|\mathbf{n}_i \cdot \mathbf{n}_j|}{|\mathbf{n}_i| \cdot |\mathbf{n}_j|} - 1 \right) \cdot |\mathbf{p}_i - \mathbf{p}_j|$ with $\mathbf{p}_i, \mathbf{p}_j$ being points on their respective axes. The relations are collected in a set $C_p$ with each of the form $g_p := \frac{1}{2} \cdot \left( (\mathbf{p}_i - \mathbf{p}_j)^T \cdot \mathbf{n}_i \right)^2 - \frac{1}{2} \cdot |\mathbf{p}_i - \mathbf{p}_j|^2 = 0$. We prune edges when deviation lengths exceed 2 \% of point set bounding box length. Similarly, for aligning a sphere with axis of a cone or a cylinder, we simply constrain the sphere center to lie on the axis of the other primitive.

Coplanar relations exist between primitive pairs, potentially non-local ones. Since we clip cylinders and cones by respective axis aligned planes, all coplanar relations in our framework reduce to detecting and conforming to coplanarity between two planes. Thus for a primitive pair $g_i$ and $g_j$, with exactly parallel normals, we add a relation $d_i = d_j$, if the normals point in the same direction, and $d_i = -d_j$, if the normals point in opposite directions. In both cases, we assign a confidence score $s_{cp} = -|d_i| - |d_j|$. The relations are appended to the set $C_p$. Such relations, even if they are few, group initially independent RANSAC primitives, e.g., plane primitives, potentially distant in space.

We sort the relations by their decreasing confidence scores and similar to the case of equal angle relations, extract a subset $C_p \subset C_p'$ by aligning to the relations $C_p \cup C_p' \cup C_k$, while minimizing the data error. During the extraction, we progressively remove low confidence relations, if necessary, until we obtain a feasible solution.

### 3.3 Equality alignment

After orientation and placement alignments, we detect and conform to equality relations. In our case, two types of equalities are detected: (i) For a pair of cylinder primitives $g_i$ and $g_j$, with comparable radii, we add a potential constraint $g_i := r_1 = r_2 = 0$ with confidence score $s_1 = -r_1 - r_2$. (ii) Relations between plane primitives and also between clipping planes, are more complicated as they involve relations between a pair of primitive pairs, similar to the equal-angle relations. For a pair of parallel plane pairs, $\{(\mathbf{n}_{12}, \mathbf{d}_1), (\mathbf{n}_{12}, \mathbf{d}_2)\}$ and $\{(\mathbf{n}_{34}, \mathbf{d}_3), (\mathbf{n}_{34}, \mathbf{d}_4)\}$, we introduce a constraint $g_i := (d_1 - d_2)^2 - (d_3 - d_4)^2 = 0$ with confidence score $s_i = -|d_1 - d_2| - |d_3 - d_4|$ (for each $g_i$, we adjust signs of $d_i$ so that all the normals point to same direction). We collect both types of relations into a set $C_c$, and as previously with equal angle case use transitivity to discard redundant relations, and extract a feasible set $C_c'$ such that the data error is minimized, while simultaneously conforming to all the constraints $C_c \cup C_p \cup C_k \cup C_i$.

### 3.4 re-RANSAC

In our framework, the optimization proceeds from a coarse to fine scale. Once the initial set of primitives are aligned using the detected global relations, we reassign the points in $P$ to the aligned primitives. The points with high data error to all the primitives are collected as unclaimed points in a set $P^*$. Additional checks for spatial consistency are not necessary since the primitives are
already clipped and bounded at this stage. We iterate through the pipeline starting with RANSAC on the unclaimed points $P^*$, with two modifications: (i) we lower the threshold on the allowed size of the smallest detected primitive based on the extent of connected components in $P^*$; (ii) we start with a relationship set of $C_{1}^* := C_{S}^* \cup C_{P}^* \cup C_{C}^* \cup C_{O}^*$, instead of an empty set. Note that for global alignment, the primitives from the previous step, and their associated data points are also incorporated. For example, in Figure 10 the small cylinders are detected in the second iteration, which are missed in the first pass due to the points from the dominant surrounding regions. Although the initial parameters for the small cylinders are unstable due to their limited support, they are stabilized by aligning to the first iteration relations $C_{i}^*$.

### 4 Evaluation

We evaluated our framework on a variety of synthetic and scanned datasets. Where precise models were available, we compared the reconstructed models with the ground truth, both in terms of recovered face normals as well as distance between parallel primitive pairs. Where precise ground truth was unavailable, we measured the physical model for comparison.

#### 4.1 Synthetic datasets

In the first experiment, we consider three Platonic solids, namely octahedron, dodecahedron, and icosahedron. In each case, we perturb the normals of the polyhedrons’ faces and then sample the model while adding noise to each sample independently. Note that the normal perturbation mimics misalignment among multiple scans (see later discussion on slippage). For each point set, we align the initial RANSAC primitives using our global optimization. Then, we compare face normals and distance between parallel primitive pairs with those in the ground truth model; we do the same for the model using initial RANSAC primitives. For a consistent comparison, we retain only the top relevant number of RANSAC planes in each case, e.g., for octahedron, we retain the top 8 RANSAC planes. Table 1 demonstrates the stability of our method across varying amount of noise.

### 4.2 Scanned datasets

We tested our method on real-world objects of varying complexity. In absence of a precise virtual model as ground truth, we used physical measurements of the objects for comparison. We also validated orientation, coplanarity, coaxial, and equality relations detected by
Figure 13: Cylinder, plane, and sphere primitives are aligned using extracted coaxis, coplanar, parallel/orthogonal axes, equal angle as well as equal length constraints. They converge to the final model after two iterations of RANSAC fitting and constraint optimization. We overlap the final result on the initial RANSAC results for comparison; the histograms demonstrate the effect in the primitive pair angle space.

in our system on the physical models, as possible.

In our simplest example of the wheel model, Figure 14, all the major axes of the planes, cylinders, and cones are detected parallel, and the reduced orientation graph is trivial in absence of any orthogonal relations. The dataset is interesting for the large number of coaxial and coplanar relations, and also for the equality of attributes for cones and cylinders. We detected all the relations (see histograms), thus producing a natural segmentation of the object based on spatial clustering of various equality relations, i.e., the corresponding subparts of the object have a common subgraph structure in the primitive pair relation graph. Note that the final model, automatically allows exploded views of the subparts, along the recovered axis directions.

Next, we take a packing foam-box (see Figures 1 and 11) with small parts sharing a variety of relations. Here the detected lengths and angles were in good agreement with the physical model. Interestingly, a protrusion on the box (right feature in Figure 1) was in the range of 0.1mm to the other flanks on (front side of) the model. All these protrusions were grouped together and made identical.

This demonstrates our global prior that if primitives are in near alignment, they are likely to be in exact alignment.

The clutch model, Figure 9, is challenging given the small features, and the poor quality of the scan, due to the dark material color (we were not allowed to paint the model white). Even on this sparse dataset, our method correctly detected various relations, hinting at the ubiquity of relations and repetitions in man-made engineering objects. Note that in one part the sample points are almost nonexistent and the method had little evidence to propagate and connect up the primitives.

In the final, but most complicated example, we scanned a machine part (see Figure 13). Various relations were identified and conformed to the final model. The angle pair histogram clearly
Figure 15: Global relations can help register multiple scans, A and B in this example, which are otherwise difficult to align simply using local registration. We first individually align each of the scans using our framework to yield aligned models A and B. Locally registering the aligned models using ICP leaves ambiguity due to rotational slippage. We remove this ambiguity using global constraints, orthogonality between yellow-blue cylindrical hole pair in this example, learned and optimized by our method. 

Figure 16: (Left) Aligning to global relations corrects for significant misaligned RANSAC primitives obtained using local fits. However, when the data quality is sufficiently poor, our algorithm may fail to recover all global relations, and hence the reconstruction is only partially correct (right).

Discussion

Although model reconstruction is not our primary goal, the aligned primitives can easily be used to extract a clean model for many man-made machine parts (see Figures 1 and 12). Using the relationship graph and the underlying data, we extrapolate and compute pairwise primitive intersections as applicable. In our current implementation, for a primitive pair involving complex primitives, e.g., cylinder with cone, we use a simple numerical method to extract a polygonal intersection curve. Our method, even under irregular sampling, yields precise models along with sharp features, which is otherwise difficult with existing alternatives. Note that by aligning to global relations in conjunction with local fitting, we demonstrate that it is possible to extract the global relations even in noisy and sparse datasets. The recovered relations are naturally a sparse encoding of the input model, where we make use of the fact that such models are of low information content. Our algorithm can be seen as precisely recovering such key relations.

Unlike our method, most scan reconstruction methods use local priors, while ignoring global relations that are dominant in man-made structures. As in any shape analysis with unreliable data, there is always a tradeoff between approximation level and extent of regularity detected. Our method allows aligning relations even at the expense of slightly increased data fitting cost. However, the user can require our method to disallow large increase in the data error, and dropping the corresponding relations. It will be interesting to explore Pareto-optimal solutions in this setting. Note that aligning to regular configurations can also help in identifying defects in scanned models, and encoding shapes as deviations from regular configurations.

A possible improvement is to consider higher-order relations involving pair of triplets (or more) of primitives, but the computational complexity quickly makes a simple generalization impractical. We leave this to future research.

Limitations

For certain types of relations, we investigate pairs of primitive pairs. This can potentially lead to $O(m^2)$ relations for $m$ primitives, especially in regular configurations, closely packed in space, resulting in significant computation times. Also when the angle or length space histograms have closely spaced values, increased noise level can submerge such differences making it difficult to correctly recover...
the relations in our framework (see Figure 8 and Table 1). Such errors can potentially accumulate across primitives.

Chance alignments involving multiple outlier primitives may produce high confidence but at the same time incurring possible undesirable relations. If such groups are large then biconnected components cannot isolate them, and subsequently these global ‘relations’ may pollute the real constraints. Finally, our approximate solution of relation group extraction, in certain cases, can miss global relations, or even introduce outlier relations that happen to have high confidence score (see Figure 16). Although we do not expect to extract the best feasible subset given the relation to the NP-complete satisfiability problem, in practice large data error can indicate such problem. In our experiments though, we did not encounter such a situation.

5 Conclusions and Future Directions

We presented a method for incorporating global relations to reliably recover relations among both local and distant parts of man-made objects. We demonstrated that for regular objects, i.e., objects with low information content, various global relations can be reliably extracted from noisy (structured) and incomplete datasets, even when the local signals are ambiguous and unreliable. Starting with a set of initial RANSAC based locally fitted primitives, our algorithm progressively learns and infers orientation, placement, and equality relations. In each stage, a set of feasible relations are extracted among the candidate relations, and then aligned while best fitting to the input data.

In the context of low complexity, procedurally modeled man-made objects, learning global priors is likely to significantly assist in shape understanding and sparse encoding. Moreover, man-made models, e.g., Figure 2-left, commonly involve hierarchical relations. Such a grouping naturally arises from manufacturing ease and reuse options. Inferring such hierarchical grouping, however, is challenging since it will involve solving a consistent segmentation across the noisy input.

In the future, we want to consider directly integrating this stage with the global alignment leading to a robust RANSAC. More precisely, partially recovered global relations can guide an importance sampling based RANSAC fitting to robustly extract primitives from locally inconsistent and sparsely sampled regions. Bringing in similar global relations to surface reconstruction remains to be explored.

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References


Table 2: Performance statistics on a laptop with Intel(R) Core(TM) 2.53GHz with 2GB, for models presented in the paper. We show the number of relations before and after reduction for the various alignment stages.


