Estimating Surface Normals in Noisy Point Cloud Data

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Abstract

We analyze a method based on local least square fitting for estimating the normals at all the sample points of a point cloud data (PCD) set, in the presence of random noise. We study the effects of neighborhood size, curvature, sampling density, and noise on the normal estimation when the PCD is sampled from a smooth curve in 2D or a smooth surface in 3D and noise is added.

The Normal Estimation Problem

Given

A noisy point cloud data (PCD) sampled from a surface (or curve) without any connectivity information

Goal

Compute surface normals at each point p

Related work

Voronoi-diagram based approach

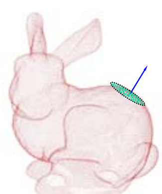
- Do a surface reconstruction (Crust, Cocone)
- Compute normals from the reconstructed surface
- Works in *absence* of noise

Surface fitting

- As found in works of Hoppe, Pauly, Alexa
- Pointshop3D, Progressive Point Set

- Use least square fit to a neighborhood of radius r around point p
- Works in *presence* of noise
- Neighborhood size picked manually

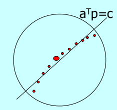
Normal Estimation using Plane Fitting



Contributions

- Study the effects of curvature, noise, sampling density on the choice of neighborhood size
- Use this insight to choose an optimal neighborhood size
- Compute bound on the estimation error

Least Square Fit

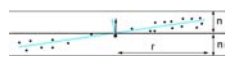


- Assume best fit surface: $a^Tp=c$
- Minimize $\frac{1}{k}\sum_{i=1}^{k}(a^T p_i - c)^2$
- Reduce to an eigenvalue problem

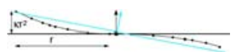
$$M = \frac{1}{k}\sum_{i=1}^{k}(p_i-\bar{p})(p_i-\bar{p})^T$$

- Smallest eigenvector of M is our estimate of the normal to the surface

Collusive Noise



Curvature Effect



Two Extremes of Deception

Analysis

- Assume, k points are uniformly distributed with density ρ within a ball of radius r
- Noise
  - independent of measurement
  - zero mean, known variance σ_n
- From the *evenly distributed* sample assumption
  - Bound all entries of the covariance matrix M
  - Important fact: Summation tends to expectation as the number of samples increases
  - Get a bound on the estimation error

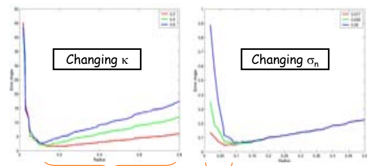
Result in 2D

$$error \le c_1\kappa r + c_2\frac{\sigma_n}{\sqrt{\varepsilon\rho r^3}} + c_3\frac{\sigma_n^2}{r^2}$$

result holds with probability $1-\varepsilon$

- This suggests, to minimize error when density is
  - high, choose optimal r as c $(\sigma_n^2/\kappa)^{1/3}$
  - low, choose optimal r as c $(\sigma_n^2/(\varepsilon\rho\kappa^2))^{1/5}$
- For flat surface, $\kappa=0$, can make error arbitrarily small
- For no noise, $\sigma_n=0$, pick as small a neighborhood as possible

2D: Simulation and Prediction



$$error \le \boxed{c_1\kappa r} + \boxed{c_2\frac{\sigma_n}{\sqrt{\varepsilon\rho r^3}} + c_3\frac{\sigma_n^2}{r^2}}$$

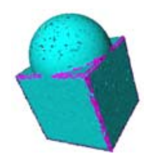
Result in 3D

- A more involved but similar analysis in 3D gives the following bound

$$error \le \theta(1)\kappa r + \theta(1)\sigma_n/(r^2\sqrt{\varepsilon\rho}) + \theta(1)\sigma_n^2/r^2$$

- So a good choice of r is given by

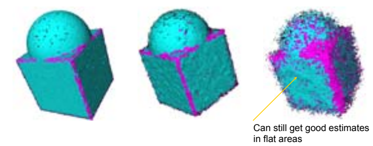
$$r = \left(\frac{1}{\kappa}\left(c_1\frac{\sigma_n}{\sqrt{\varepsilon\rho}} + c_2\sigma_n^2\right)\right)^{1/3}$$



Exact Model used to Choose c_1, c_2

Estimation Error > 5°

Effects of Curvature, Noise on Neighborhood Size



1x noise　2x noise　4x noise

Effect of Increasing Noise on Estimation Error

Algorithm for Normal Estimation in 3D

- Choose c_1, c_2
- Initialize k=15
- Iterate and refine (10 steps or less)
  - Compute r, ρ, κ locally
    - $r \approx$ distance to k-th nearest neighbor
    - $\rho = k/\pi r^2$
    - κ computed using Gumhold's method
  - Use ρ, κ to estimate r_{new}
  - Re-estimate

$$k_{new} = \pi r_{new}^2 \rho_{old}$$

  - Stop if
    - k > threshold or,
    - k saturates
- Use least square fit on the k-nearest neighbors to get the normal estimate to the surface at p

Color-coded Neighborhood Size

Estimation Error on Happy Buddha

More information: *Estimating Surface Normals in Noisy Point Cloud Data*, Niloy J. Mitra, An Nguyen, SoCG 2003 (to appear)