

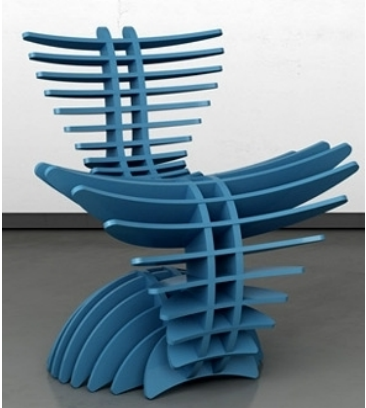
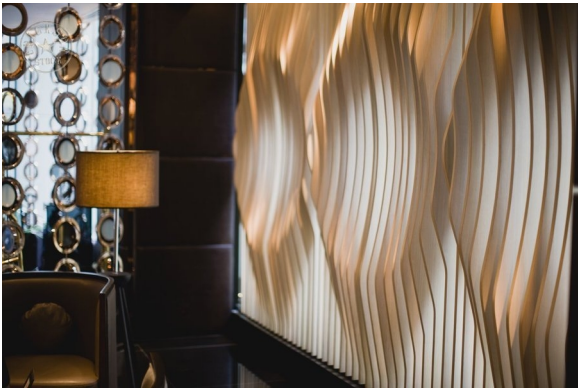
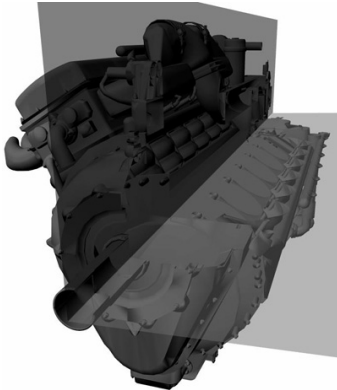
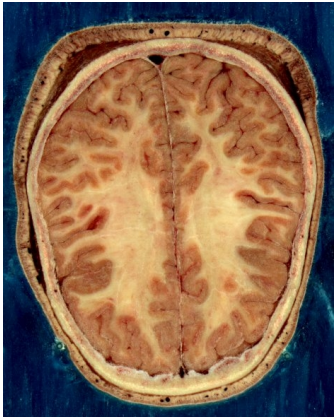
Surface Perception of Planar Abstractions

James McCrae
Niloy J. Mitra
Karan Singh

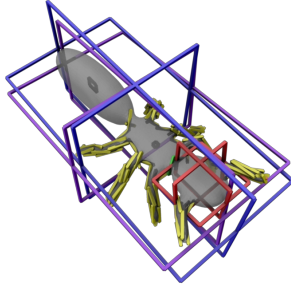
dgp | Dynamic Graphics Project
University of Toronto
www.dgp.toronto.edu



Planar Abstractions



Planar Abstractions



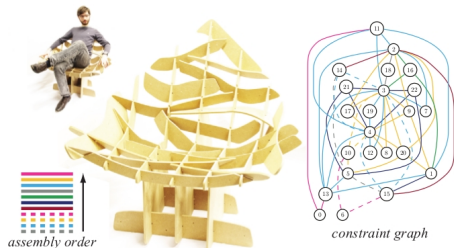
Slices: A Shape-proxy Based on Planar Sections

James McCrae, Karan Singh, Niloy Mitra
SIGGRAPH Asia, 2011



crdbrd: Shape Fabrication by Sliding Planar Slices

Kristian Hildebrand, Bernd Bickel, Marc Alexa
Eurographics, 2012

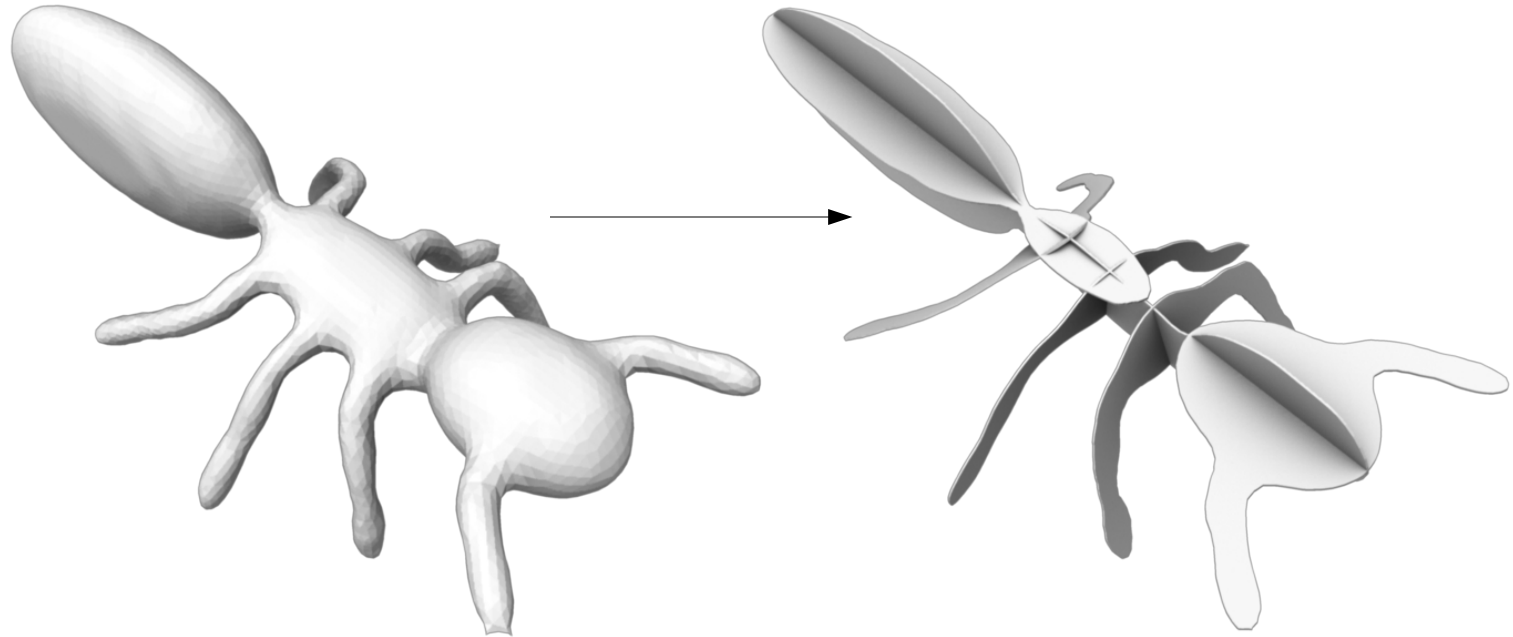


Fabrication-aware Design with Intersecting Planar Pieces

Yuliy Schwartzburg and Mark Pauly
Eurographics, 2013

Planar Abstractions

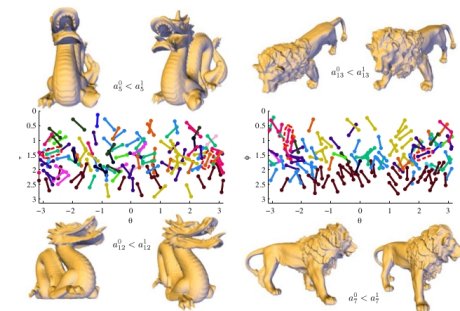
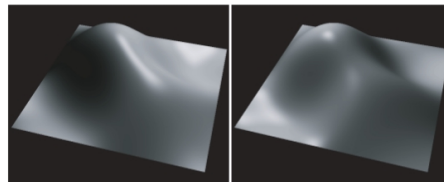
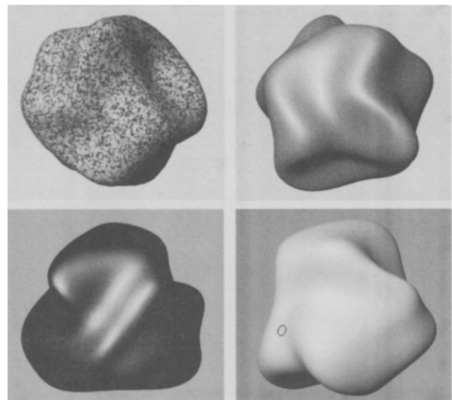
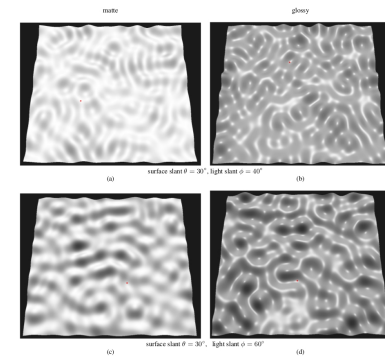
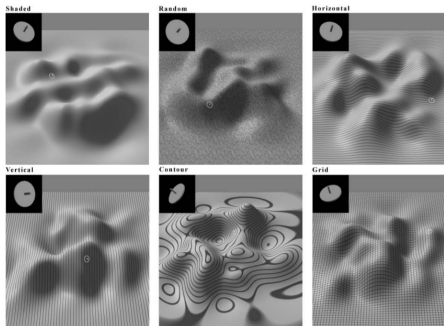
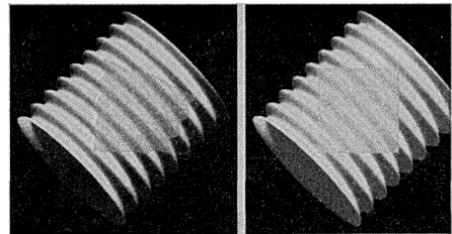
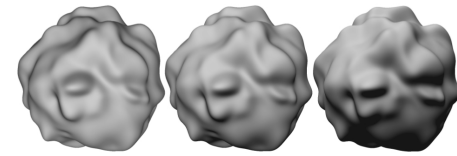
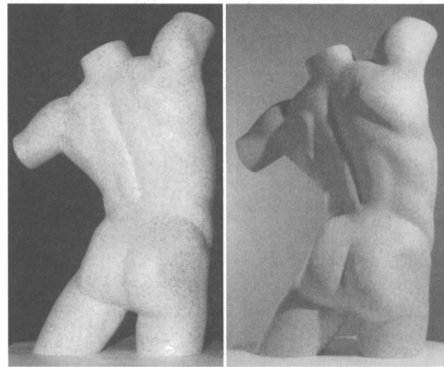
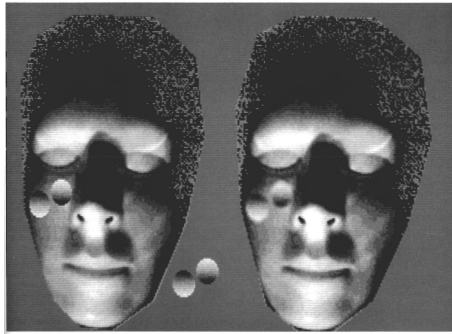
How effective are planar abstractions at representing shape?



Input surface

Planar abstraction

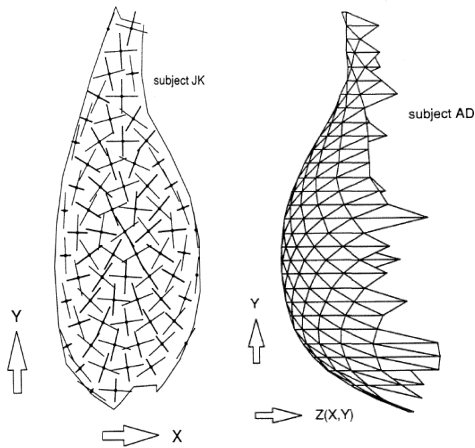
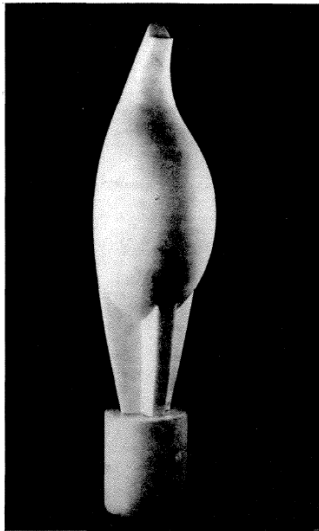
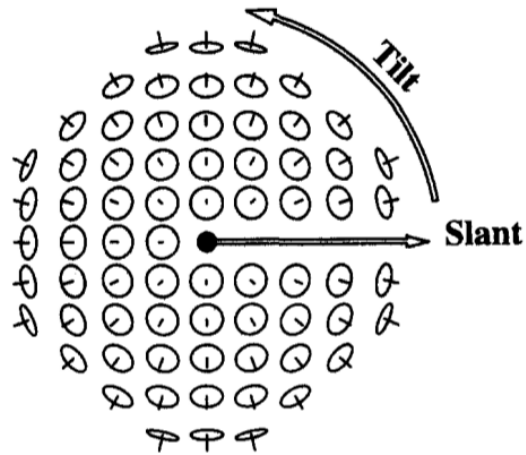
Visual Perception of Shape



Visual Perception of Shape

Surface Perception in Pictures

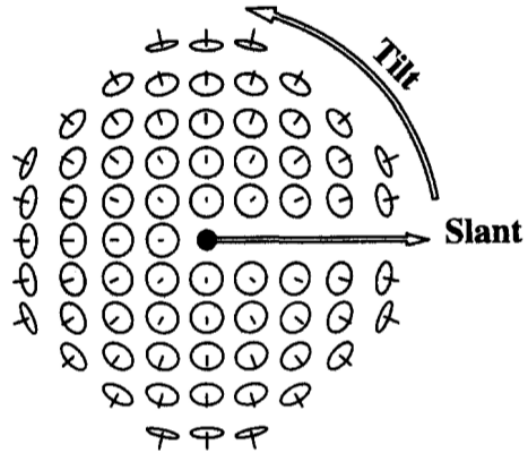
Jan J. Koenderink, Andrea J. van Doorn,
Astrid M. L. Kappers
Perception and Psychophysics, 1992



Visual Perception of Shape

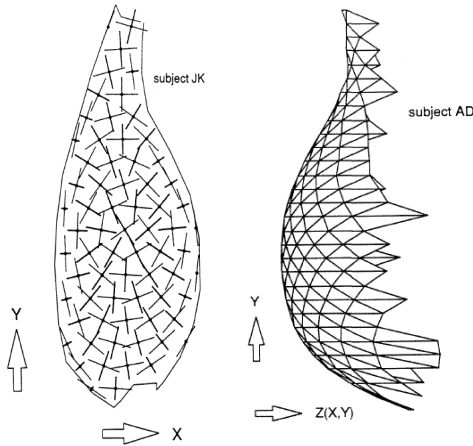
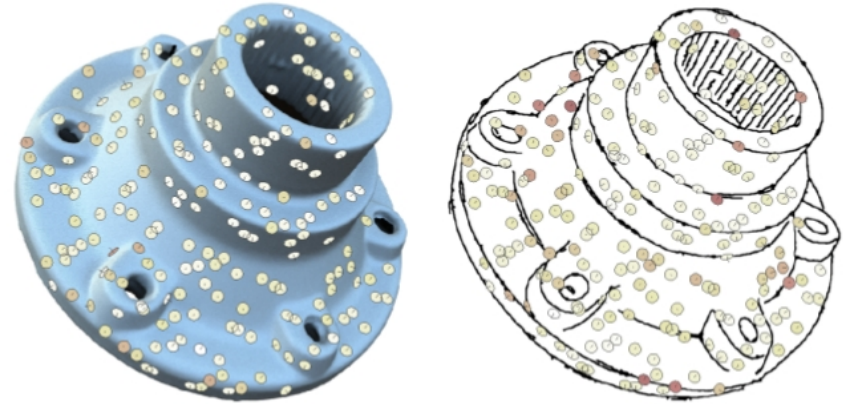
Surface Perception in Pictures

Jan J. Koenderink, Andrea J. van Doorn,
Astrid M. L. Kappers
Perception and Psychophysics, 1992



How well do line drawings depict shape?

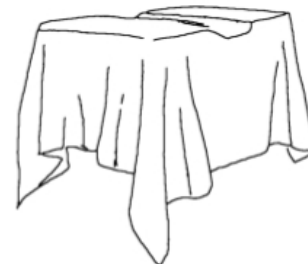
Forrester Cole, Kevin Sanik, Doug DeCarlo, Adam Finkelstein,
Thomas Funkhouser, Szymon Rusinkiewicz, Manish Singh
ACM Transactions on Graphics (Proc. SIGGRAPH), 2009



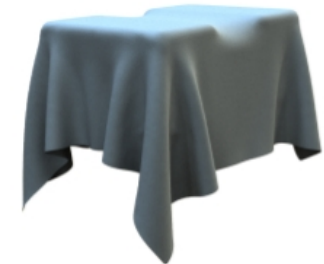
ridges and valleys



suggestive contours



artist's drawing



shaded

Overview

Surface perception user study

Analysis of participant study data

Predicting perception error and application

Overview

Surface perception user study

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Predicting perception error and application

User Study

5 representations

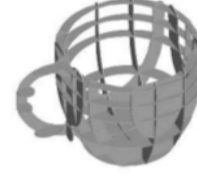


User Study

5 representations



Surface

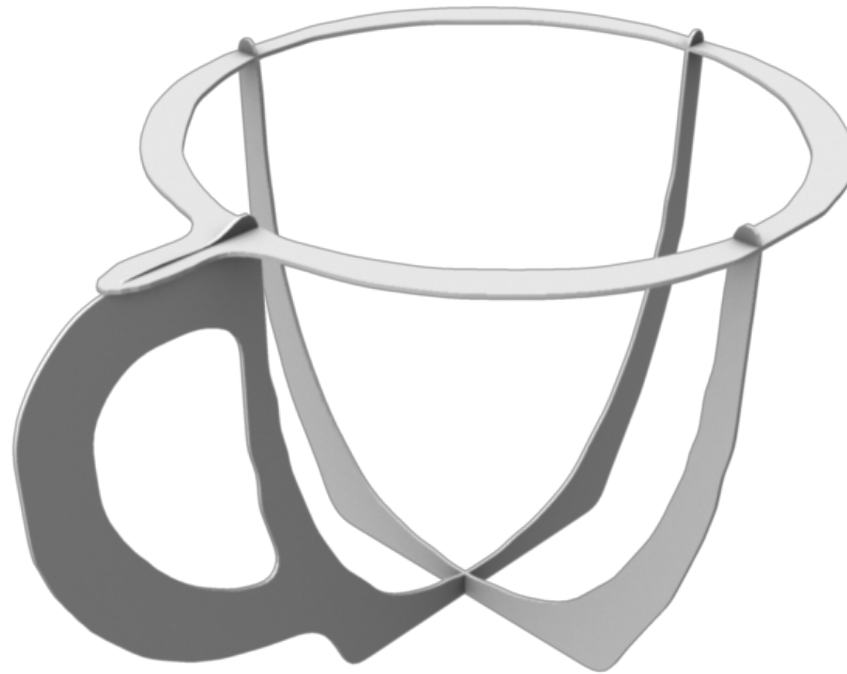


User Study

5 representations



MPS*



**Slices: A Shape-proxy Based on Planar Sections*

James McCrae, Karan Singh, Niloy Mitra
SIGGRAPH Asia, 2011

User Study

5 representations



XYZ



User Study

5 representations

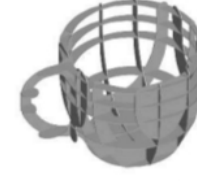


radial

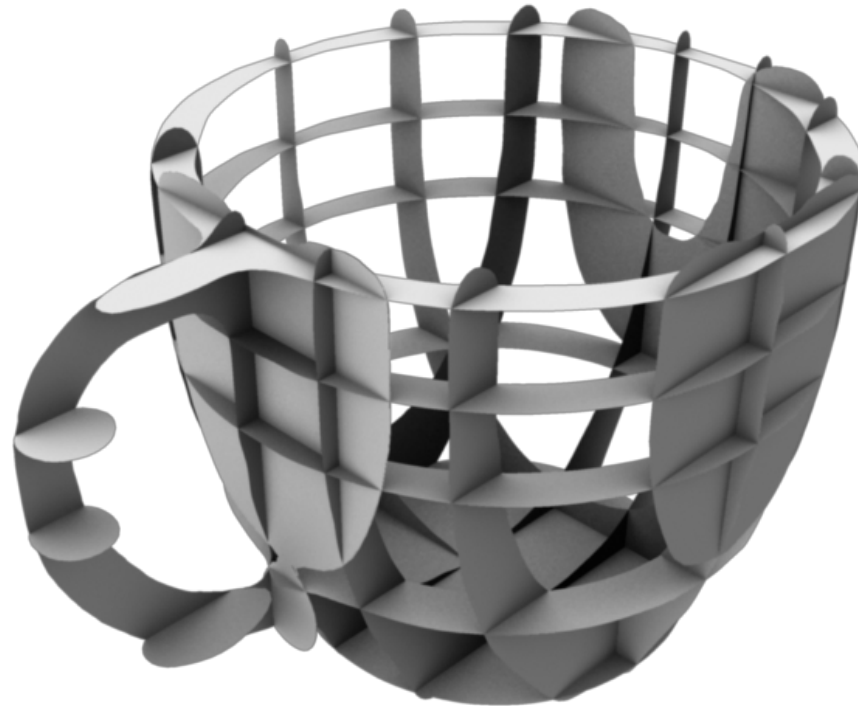


User Study

5 representations

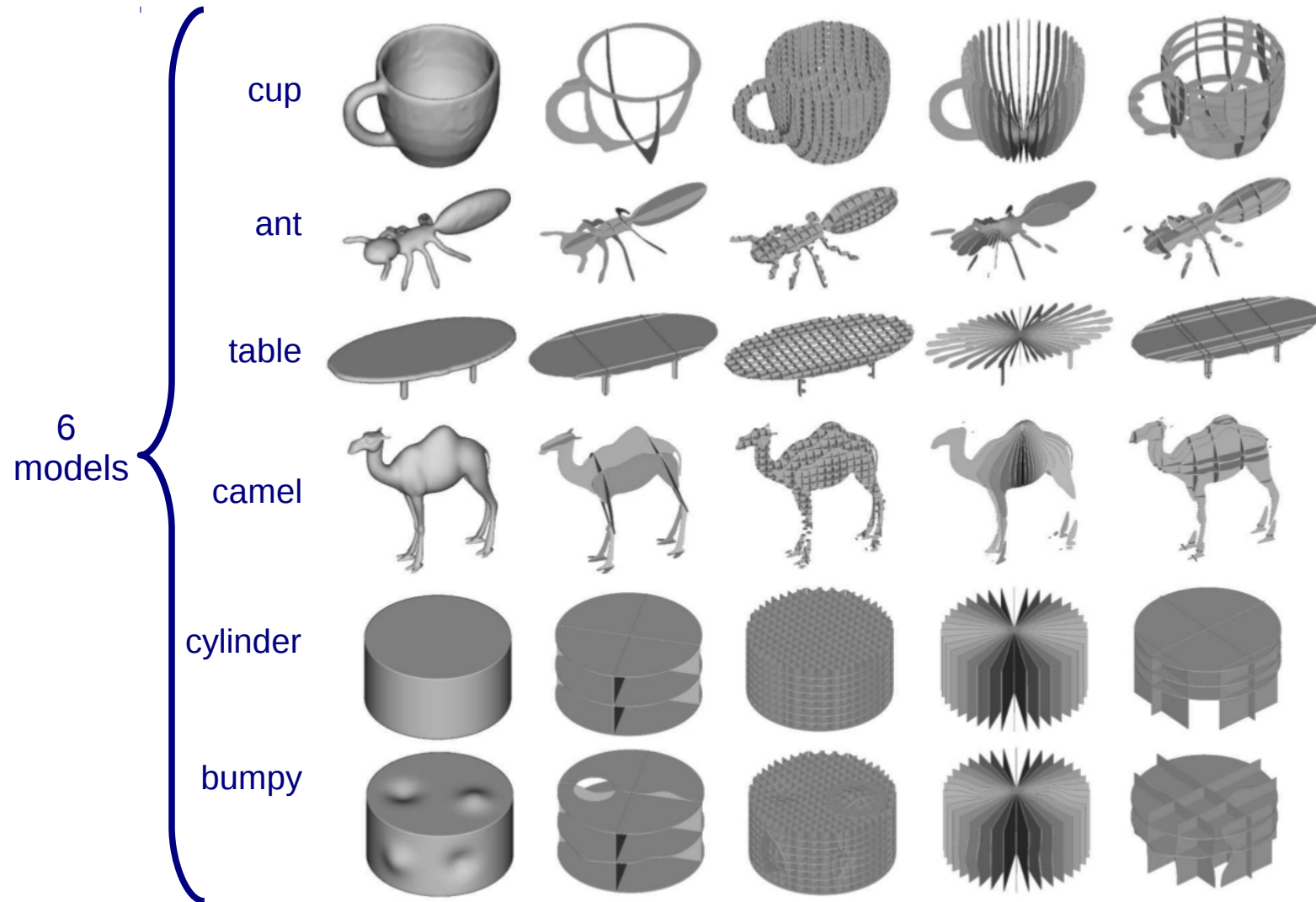


crdbrd*



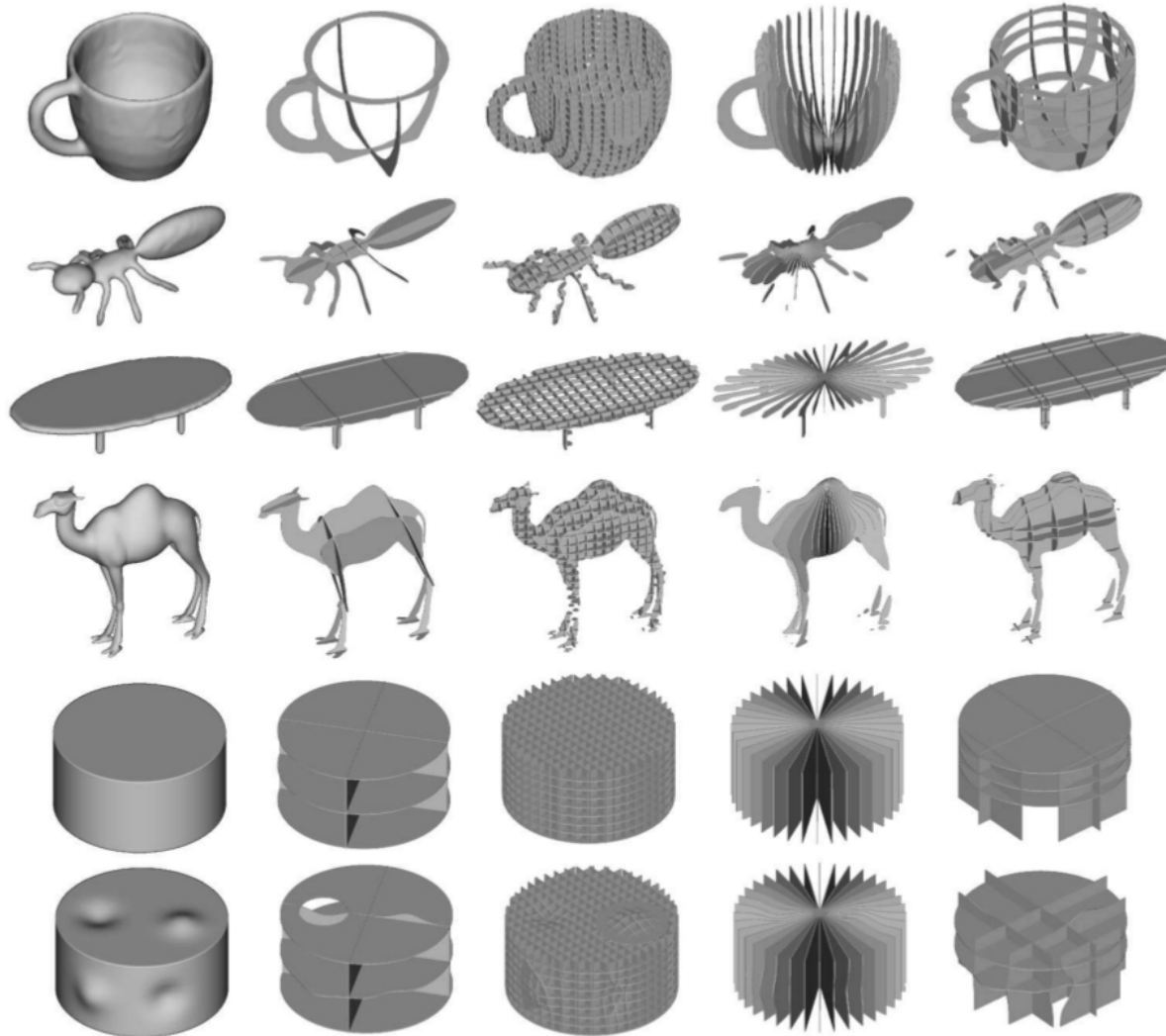
**crdbrd: Shape Fabrication by Sliding Planar Slices*
Kristian Hildebrand, Bernd Bickel, Marc Alexa
Eurographics, 2012

User Study



User Study

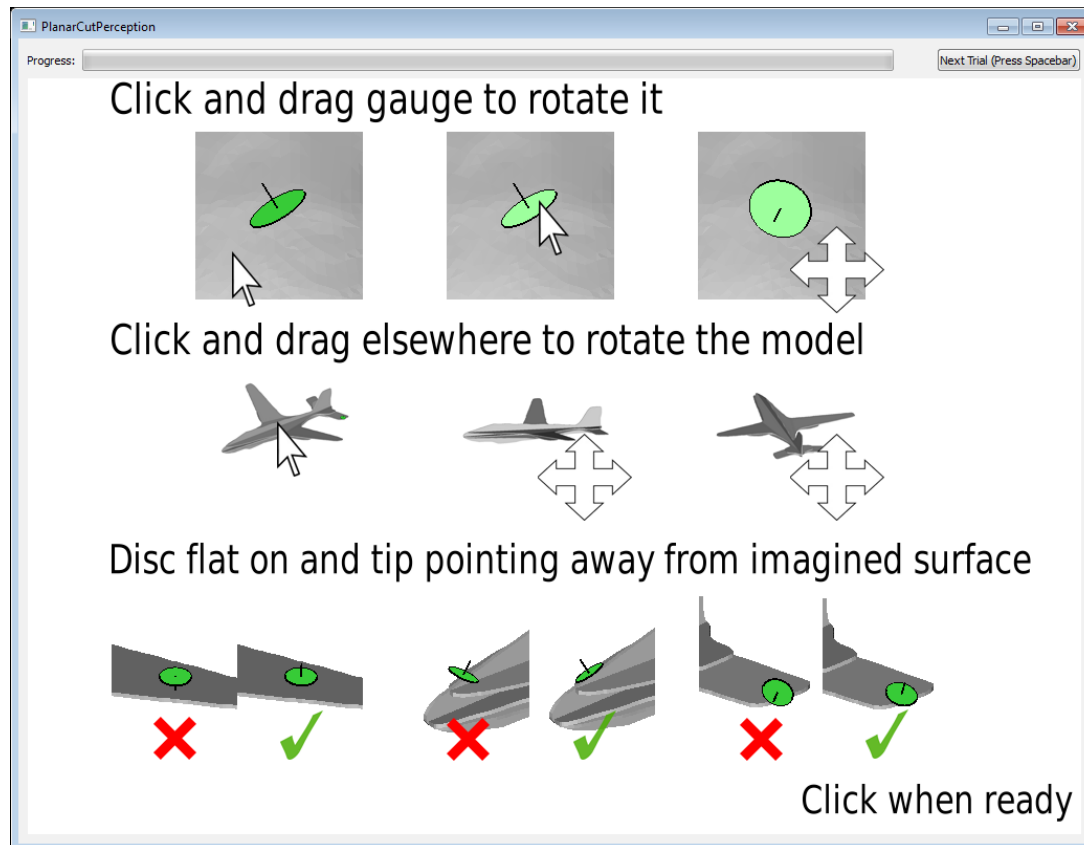
5 representations * 6 models = 30 different visual stimuli in total



User Study

Methodology

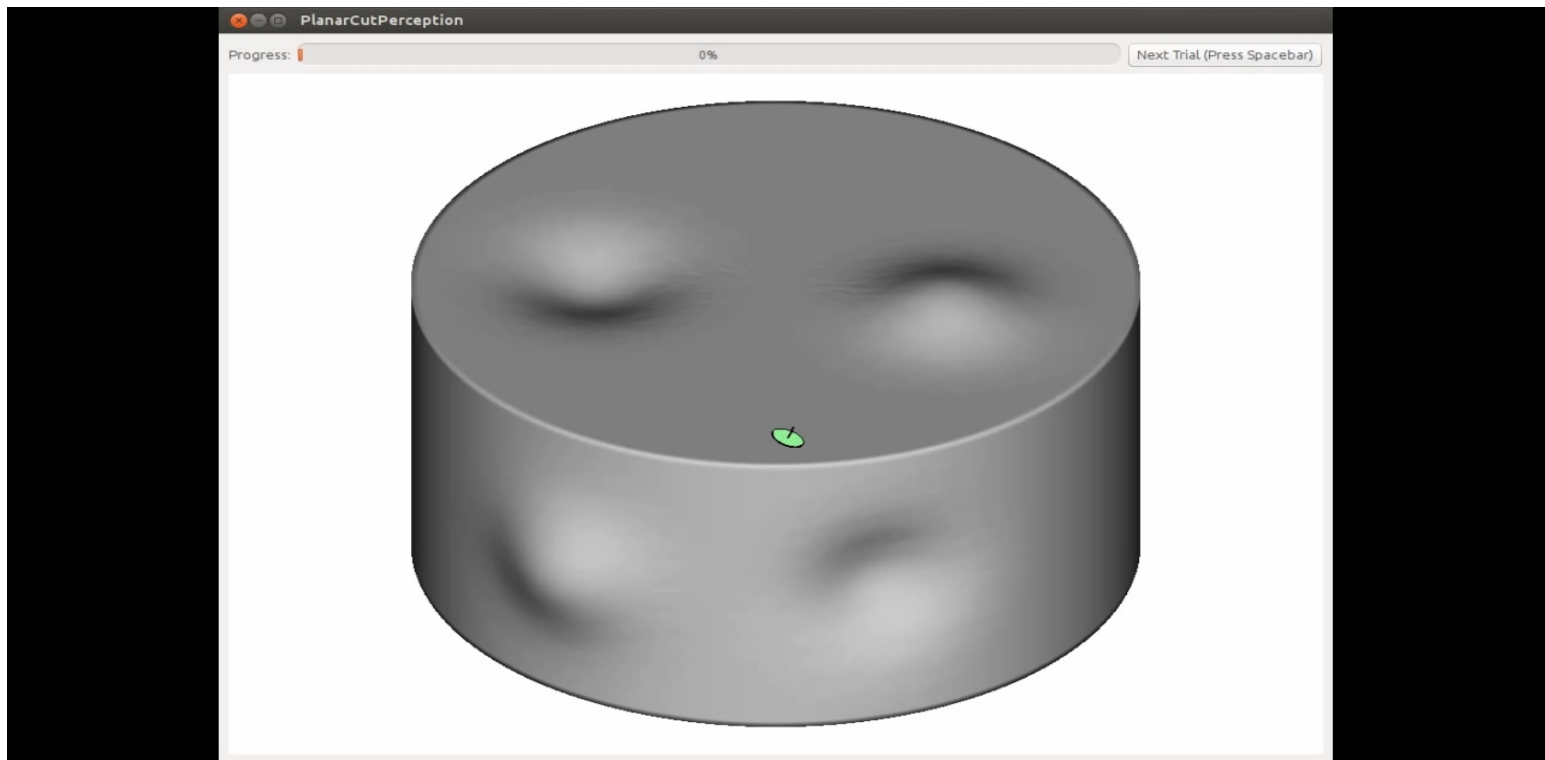
Mechanical Turk participants with a *unique worker ID* view an instructions screen.



User Study

Methodology

Mechanical Turk participants with a *unique worker ID* view an *instructions screen*. Participants adjust 60 gauges (30 pairs).

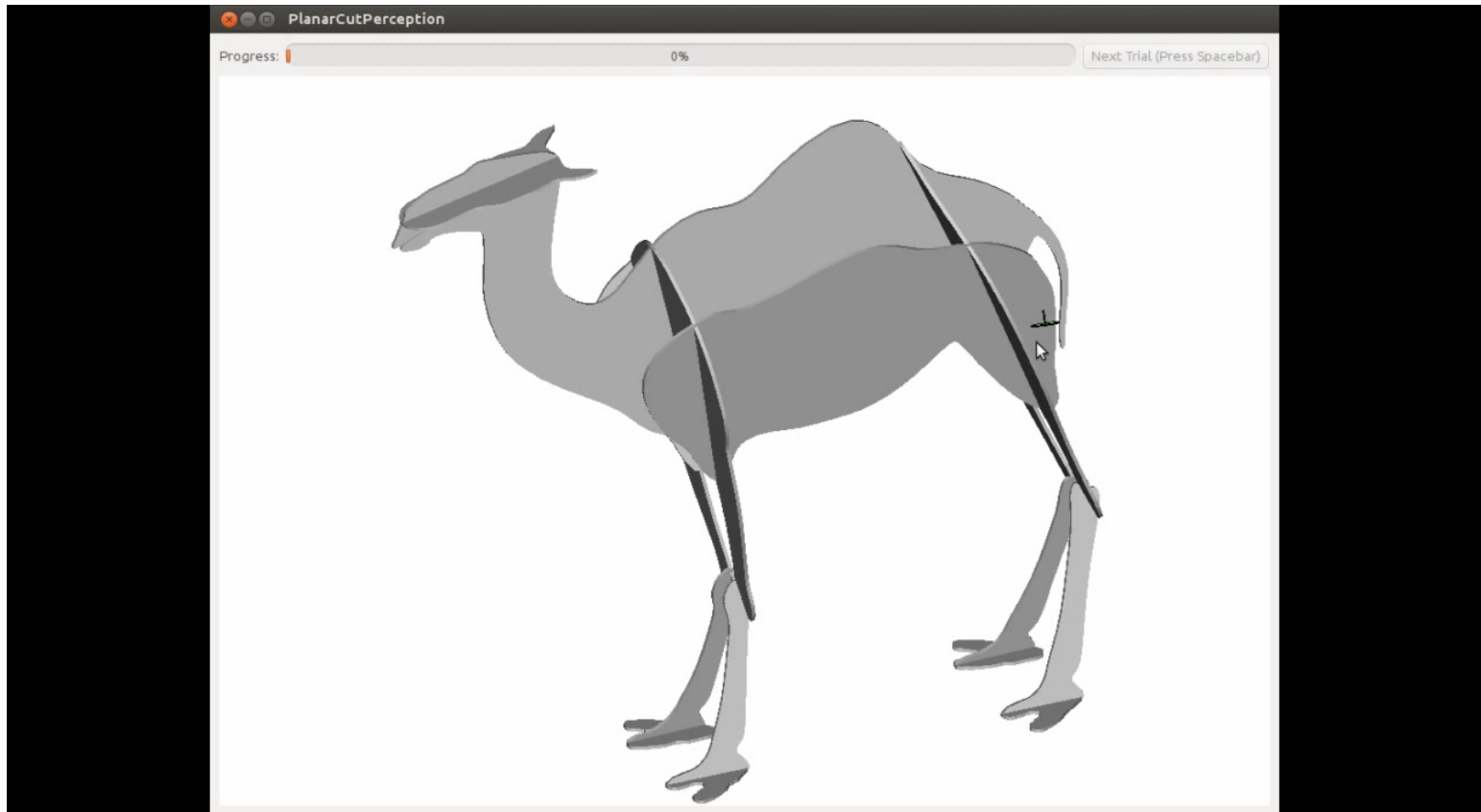


Conditions in video: fixed view task, surface representation

User Study

Methodology

Mechanical Turk participants with a *unique worker ID* view an *instructions screen*. Participants adjust 60 gauges (30 pairs).



Conditions in video: rotated view task, MPS representation

User Study

Data Collection

- 178 unique participants
- 1161 runs of the study
- \$0.35 paid for each run
- Total of ~70,000 gauge samples

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Data Verification

- Participants are greedy (want most money in shortest time/least effort)

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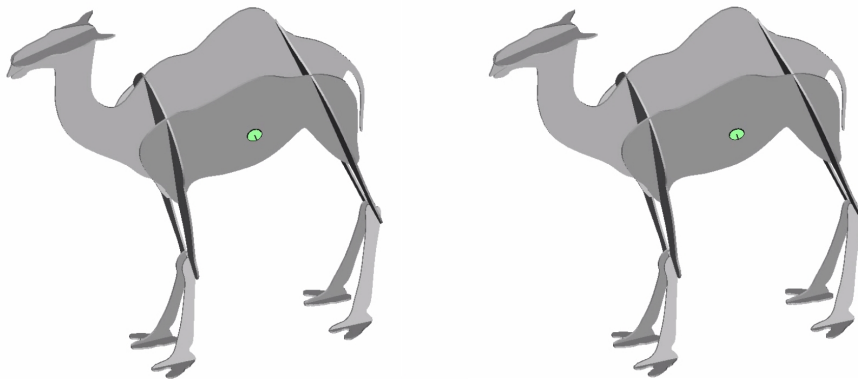
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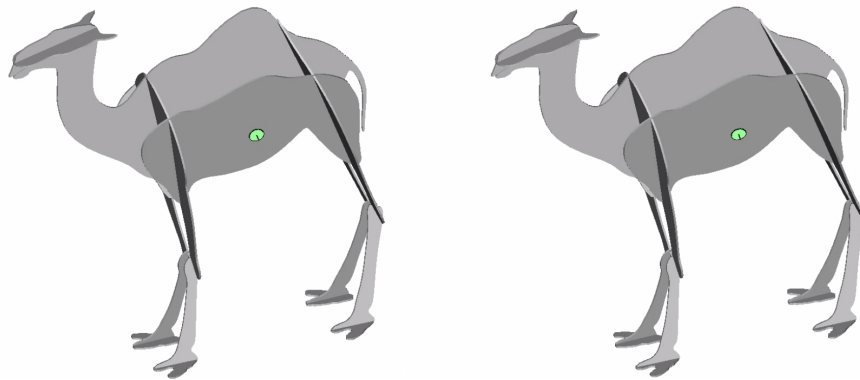
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Conditions for angle between gauge pairs

- < 30 degrees, 70% of the time
- standard deviation > 5 degrees

Overview

Surface perception user study

Analysis of participant study data

Predicting perception error and application

Analysis





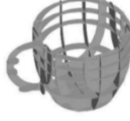




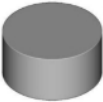

Initial Analysis

- Outliers (removal if mean error with group > 3 standard deviations)
 - One participant with mean error > 120 degrees
 - 4 of 182 participants classified as outliers, removed

Analysis

Initial Analysis





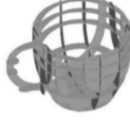




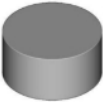

- Outliers (removal if mean error with group > 3 standard deviations)
 - One participant with mean error > 120 degrees
 - 4 of 182 participants in total classified as outliers and removed
- Average error

						
	surface	MPS	XYZ	radial	crdbrd	
 Cup	20.7	50.4	31.5	62.7	54.7	
	22.5	46.7	23.9	44.4	55.2	
 Ant	24.6	40.8	32.5	44.9	48.0	
	24.5	39.9	30.8	38.0	47.9	
 Table	13.2	14.8	17.4	49.9	16.8	
	13.1	15.0	13.4	38.5	26.0	
 Camel	26.1	35.9	28.5	42.2	40.9	
	25.5	34.3	25.7	34.6	44.9	
 Cylinder	17.7	36.8	21.1	58.3	25.7	
	16.2	35.2	15.6	41.5	31.4	
 Bumpy	19.2	38.3	20.8	59.2	42.0	
	18.8	35.5	16.3	42.3	40.9	
Fixed view	Average	20.2	36.2	24.3	52.8	38.0
Rotated view		20.1	34.4	21.0	39.9	41.0

Analysis

Initial Analysis


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		surface	MPS	XYZ	radial	crdbrd
	Cup	20.7	50.4	31.5	62.7	54.7
		22.5	46.7	23.9	44.4	55.2
	Ant	24.6	40.8	32.5	44.9	48.0
		24.5	39.9	30.8	38.0	47.9
	Table	13.2	14.8	17.4	49.9	16.8
		13.1	15.0	13.4	38.5	26.0
	Camel	26.1	35.9	28.5	42.2	40.9
		25.5	34.3	25.7	34.6	44.9
	Cylinder	17.7	36.8	21.1	58.3	25.7
		16.2	35.2	15.6	41.5	31.4
	Bumpy	19.2	38.3	20.8	59.2	42.0
		18.8	35.5	16.3	42.3	40.9
Fixed view	Average	20.2	36.2	24.3	52.8	38.0
Rotated view		20.1	34.4	21.0	39.9	41.0

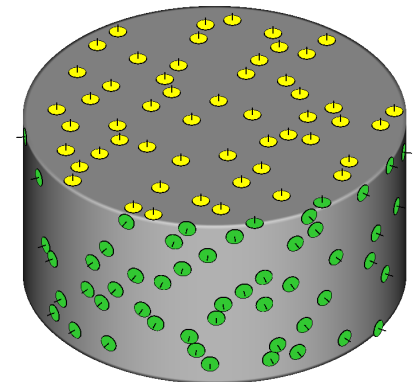
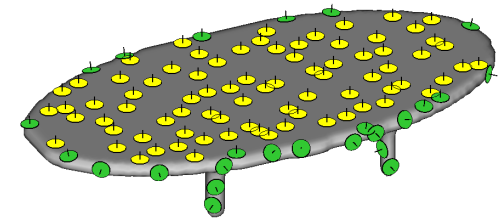
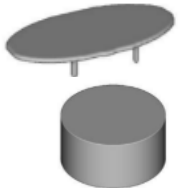
Analysis

Initial Analysis

- Outliers (removal if mean error with group > 3 standard deviations)
 - One participant with mean error > 120 degrees
 - 4 of 182 participants in total classified as outliers and removed
- Average error
- **Average error (for flat models/regions only)**



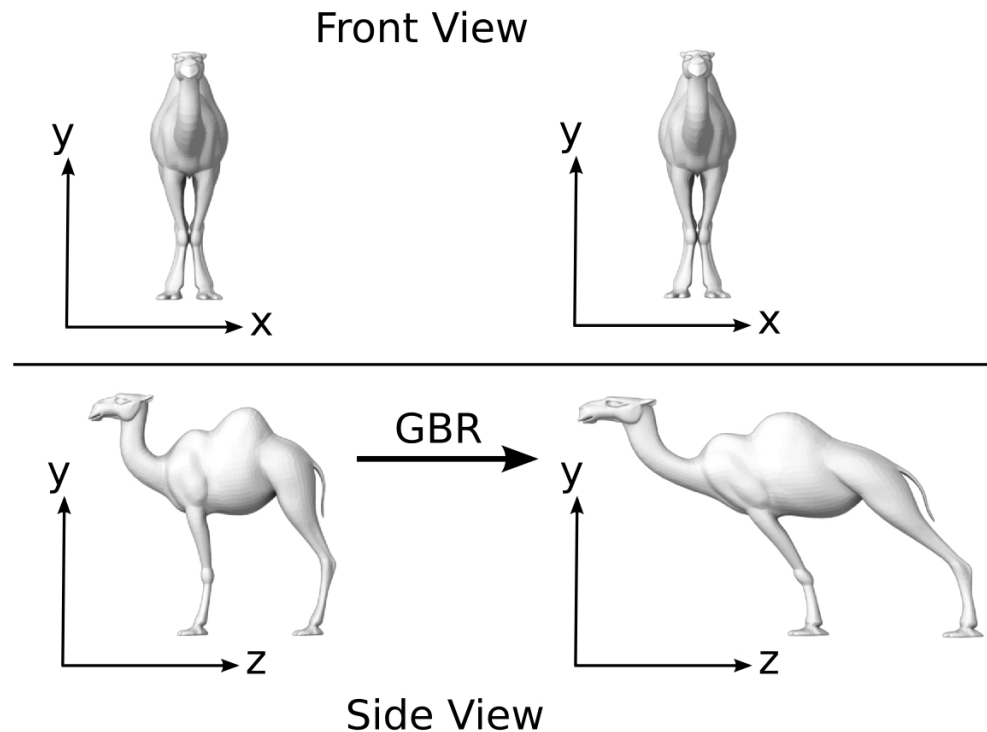
	surface	MPS	XYZ	radial	crdbrd
Table	10.6	10.9	12.7	48.8	12.5
	9.2	10.5	8.3	37.1	22.2
Cylinder	15.5	10.6	14.4	48.2	11.6
	13.3	9.9	7.9	34.9	19.1
Average	13.1	10.7	13.5	48.5	12.0
Rotated view	11.2	10.2	8.1	36.0	20.7



Analysis

Initial Analysis

- Outliers (removal if mean error with group > 3 standard deviations)
 - One participant with mean error > 120 degrees
 - 4 of 182 participants in total classified as outliers and removed
- Average error
- Average error (for flat models/regions only)
- **On the bas-relief ambiguity**



Analysis

Initial Analysis

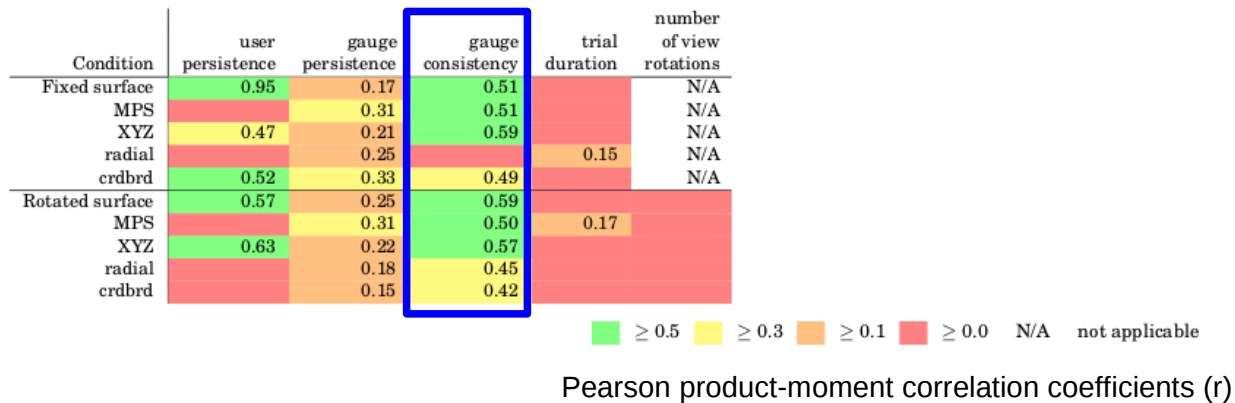
- Outliers (removal if mean error with group > 3 standard deviations)
 - One participant with mean error > 120 degrees
 - 4 of 182 participants in total classified as outliers and removed
- Average error
- Average error (for flat models/regions only)
- On the bas-relief ambiguity
 - Does not apply to rotated view task
 - Fixed view and rotated view tasks had same performance
 - Applying optimal GBR transform can significantly reduce error (often more than 5 degrees)

Fixed view	Average	20.2	36.2	24.3	52.8	38.0
Rotated view		20.1	34.4	21.0	39.9	41.0

Analysis

General correlations

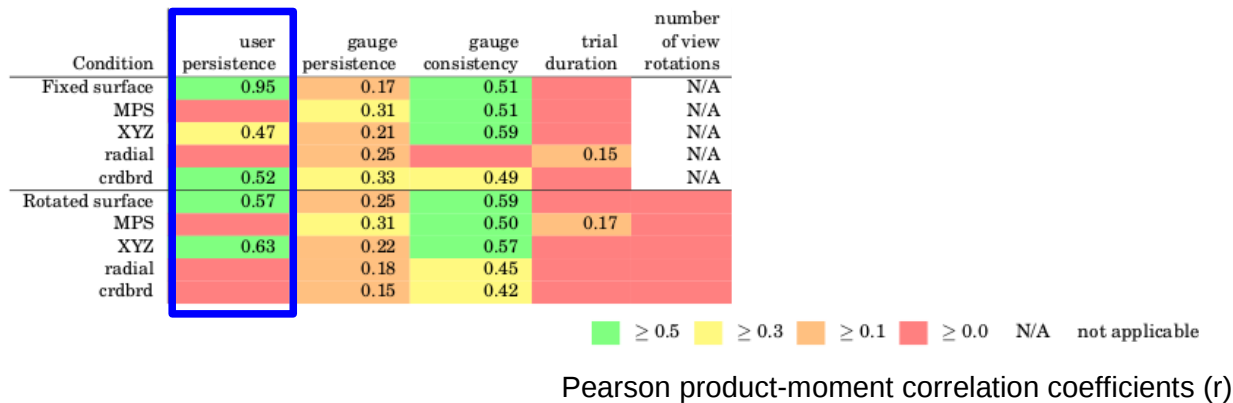
- Gauge consistency (participant agreement)



Analysis

General correlations

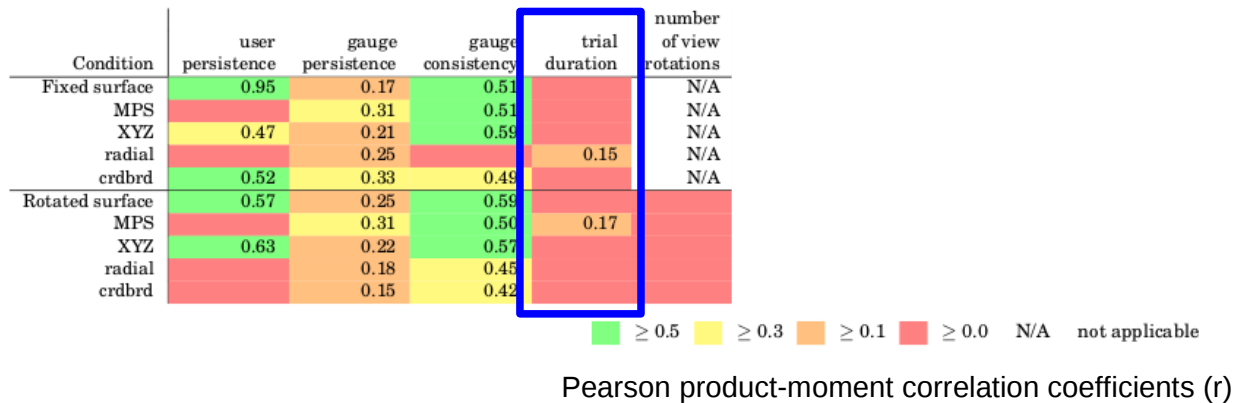
- Gauge consistency (participant agreement)
- User persistence (a participant's gauge pair settings match)



Analysis

General correlations

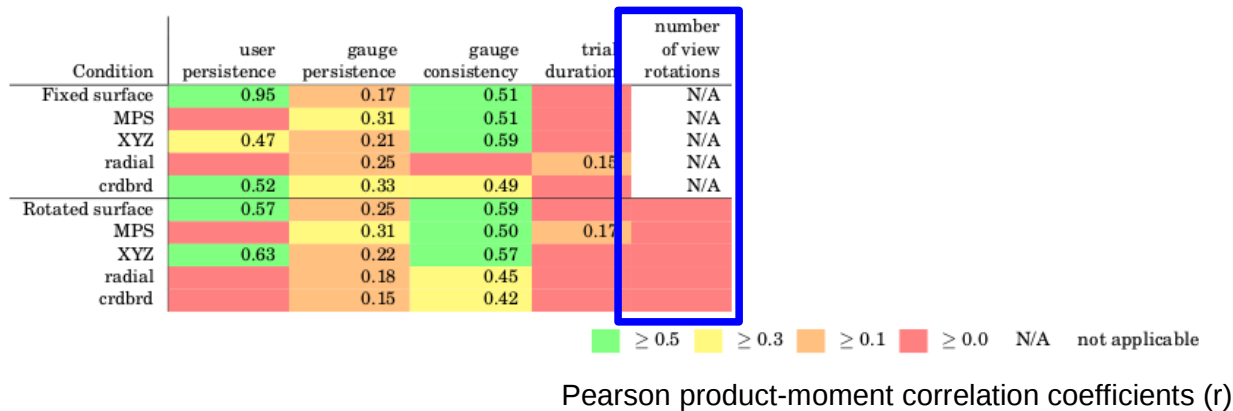
- Gauge consistency (participant agreement)
- User persistence (a participant's gauge pair settings match)
- Gauge persistence, trial duration, number of view rotations



Analysis

General correlations

- Gauge consistency (participant agreement)
- User persistence (a participant's gauge pair settings match)
- Gauge persistence, trial duration, number of view rotations



Analysis

Surface-specific correlations

- Curvature (κ_1 , κ_2 , Gaussian, mean)

Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations	absolute κ_1	absolute κ_2	Gaussian curvature	mean curvature	local thickness	medial axis distance	centroid distance	view-norm angle difference
Fixed surface	0.95	0.17	0.51		N/A	0.51	0.42	0.44	0.49		-0.13		-0.17
MPS		0.31	0.51		N/A	0.42	0.26	0.19	0.42	0.49	-0.20	0.23	
XYZ	0.47	0.21	0.59		N/A	0.54	0.42	0.46	0.53		-0.24	0.11	-0.14
radial		0.25		0.15	N/A		-0.14	-0.16		0.39	-0.10	0.28	0.21
crdbrd	0.52	0.33	0.49		N/A	0.46	0.41	0.33	0.46	0.14	-0.16	0.15	
Rotated surface	0.57	0.25	0.59			0.60	0.50	0.53	0.57		-0.21		-0.19
MPS		0.31	0.50	0.17		0.40	0.25	0.18	0.39	0.46	-0.20	0.21	-0.13
XYZ	0.63	0.22	0.57			0.64	0.50	0.56	0.62		-0.28		-0.22
radial		0.18	0.45			0.12			0.11	0.22	-0.16	0.21	0.15
crdbrd		0.15	0.42			0.43	0.37	0.31	0.42		-0.20	0.10	

■ ≥ 0.5
■ ≥ 0.3
■ ≥ 0.1
■ ≥ 0.0
 N/A not applicable

Pearson product-moment correlation coefficients (r)

Analysis

Surface-specific correlations

- Curvature (κ_1 , κ_2 , Gaussian, mean)
- Local thickness

Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations	absolute κ_1	absolute κ_2	Gaussian curvature	mean curvature	local thickness	medial axis distance	centroid distance	view-norm angle difference
Fixed surface	0.95	0.17	0.51		N/A	0.51	0.42	0.44	0.49		-0.13		-0.17
MPS		0.31	0.51		N/A	0.42	0.26	0.19	0.42	0.49	-0.20	0.23	
XYZ	0.47	0.21	0.59		N/A	0.54	0.42	0.46	0.53		-0.24	0.11	-0.14
radial		0.25		0.15	N/A		-0.14	-0.16		0.39	-0.10	0.28	0.21
crdbrd	0.52	0.33	0.49		N/A	0.46	0.41	0.33	0.46	0.14	-0.16	0.15	
Rotated surface	0.57	0.25	0.59			0.60	0.50	0.53	0.57		-0.21		-0.19
MPS		0.31	0.50	0.17		0.40	0.25	0.18	0.39	0.46	-0.20	0.21	-0.13
XYZ	0.63	0.22	0.57			0.64	0.50	0.56	0.62		-0.28		-0.22
radial		0.18	0.45			0.12			0.11	0.22	-0.16	0.21	0.15
crdbrd		0.15	0.42			0.43	0.37	0.31	0.42		-0.20	0.10	

■ ≥ 0.5
■ ≥ 0.3
■ ≥ 0.1
■ ≥ 0.0
 N/A not applicable

Pearson product-moment correlation coefficients (r)

Analysis

Surface-specific correlations

- Curvature (κ_1 , κ_2 , Gaussian, mean)
- Local thickness
- Medial axis distance, centroid distance, view-norm angle difference

Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations	absolute κ_1	absolute κ_2	Gaussian curvature	mean curvature	local thickness	medial axis distance	centroid distance	view-norm angle difference
Fixed surface	0.95	0.17	0.51		N/A	0.51	0.42	0.44	0.49		-0.13		-0.17
MPS		0.31	0.51		N/A	0.42	0.26	0.19	0.42	0.49	-0.20	0.23	
XYZ	0.47	0.21	0.59		N/A	0.54	0.42	0.46	0.53		-0.24	0.11	-0.14
radial		0.25		0.15	N/A		-0.14	-0.16		0.39	-0.10	0.28	0.21
crdbrd	0.52	0.33	0.49		N/A	0.46	0.41	0.33	0.46	0.14	-0.16	0.15	
Rotated surface	0.57	0.25	0.59			0.60	0.50	0.53	0.57		-0.21		-0.19
MPS		0.31	0.50	0.17		0.40	0.25	0.18	0.39	0.46	-0.20	0.21	-0.13
XYZ	0.63	0.22	0.57			0.64	0.50	0.56	0.62		-0.28		-0.22
radial		0.18	0.45			0.12			0.11	0.22	-0.16	0.21	0.15
crdbrd		0.15	0.42			0.43	0.37	0.31	0.42		-0.20	0.10	

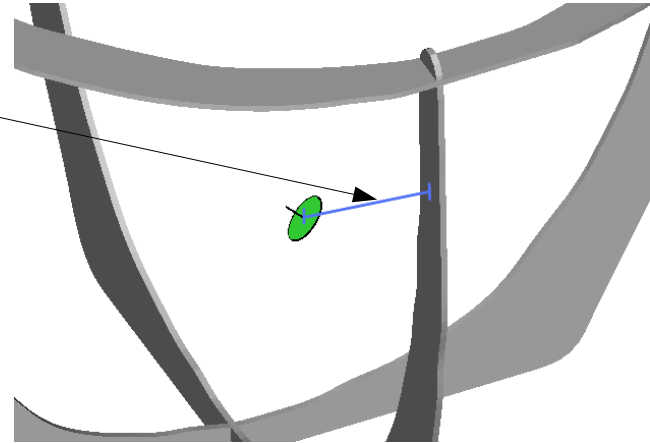
■ ≥ 0.5
■ ≥ 0.3
■ ≥ 0.1
■ ≥ 0.0
 N/A not applicable

Pearson product-moment correlation coefficients (r)

Analysis

Abstraction-specific correlations

- Abstraction distance



Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations	absolute κ_1	absolute κ_2	Gaussian curvature	mean curvature	local thickness	medial axis distance	centroid distance	view-norm angle difference	abstraction distance	abstraction angle difference
Fixed surface	0.95	0.17	0.51		N/A	0.51	0.42	0.44	0.49		-0.13		-0.17	N/A	N/A
MPS		0.31	0.51		N/A	0.42	0.26	0.19	0.42	0.49	-0.20	0.23		0.39	0.72
XYZ	0.47	0.21	0.59		N/A	0.54	0.42	0.46	0.53		-0.24	0.11	-0.14	0.13	-0.26
radial		0.25		0.15	N/A		-0.14	-0.16		0.39	-0.10	0.28	0.21	0.20	0.30
crdbrd	0.52	0.33	0.49		N/A	0.46	0.41	0.33	0.46	0.14	-0.16	0.15		0.17	0.43
Rotated surface	0.57	0.25	0.59			0.60	0.50	0.53	0.57		-0.21		-0.19	N/A	N/A
MPS		0.31	0.50	0.17		0.40	0.25	0.18	0.39	0.46	-0.20	0.21	-0.13	0.37	0.72
XYZ	0.63	0.22	0.57			0.64	0.50	0.56	0.62		-0.28		-0.22		-0.35
radial		0.18	0.45			0.12			0.11	0.22	-0.16	0.21	0.15	0.19	0.17
crdbrd		0.15	0.42			0.43	0.37	0.31	0.42		-0.20	0.10			0.43

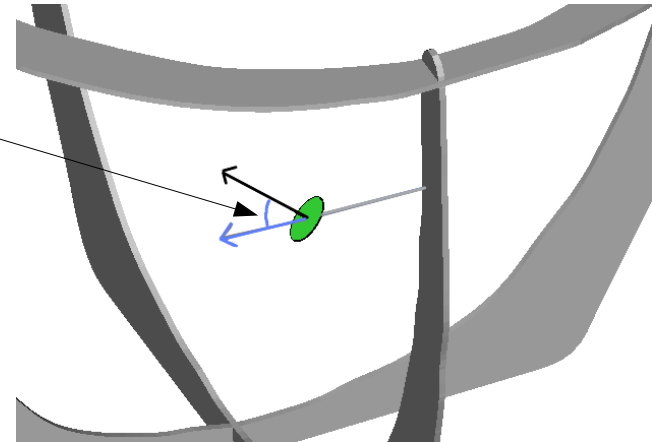
■ ≥ 0.5
■ ≥ 0.3
■ ≥ 0.1
■ ≥ 0.0
 N/A not applicable

Pearson product-moment correlation coefficients (r)

Analysis

Abstraction-specific correlations

- Abstraction distance
- Abstraction angle difference



Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations	absolute κ_1	absolute κ_2	Gaussian curvature	mean curvature	local thickness	medial axis distance	centroid distance	view-norm angle difference	abstraction distance	abstraction angle difference
Fixed surface	0.95	0.17	0.51		N/A	0.51	0.42	0.44	0.49		-0.13		-0.17	N/A	N/A
MPS		0.31	0.51		N/A	0.42	0.26	0.19	0.42	0.49	-0.20	0.23		0.39	0.72
XYZ	0.47	0.21	0.59		N/A	0.54	0.42	0.46	0.53		-0.24	0.11	-0.14	0.13	-0.26
radial		0.25		0.15	N/A		-0.14	-0.16		0.39	-0.10	0.28	0.21	0.20	0.30
crdbrd	0.52	0.33	0.49		N/A	0.46	0.41	0.33	0.46	0.14	-0.16	0.15		0.17	0.43
Rotated surface	0.57	0.25	0.59			0.60	0.50	0.53	0.57		-0.21		-0.19	N/A	N/A
MPS		0.31	0.50	0.17		0.40	0.25	0.18	0.39	0.46	-0.20	0.21	-0.13	0.37	0.72
XYZ	0.63	0.22	0.57			0.64	0.50	0.56	0.62		-0.28		-0.22		-0.35
radial		0.18	0.45			0.12			0.11	0.22	-0.16	0.21	0.15	0.19	0.17
crdbrd		0.15	0.42			0.43	0.37	0.31	0.42		-0.20	0.10			0.43

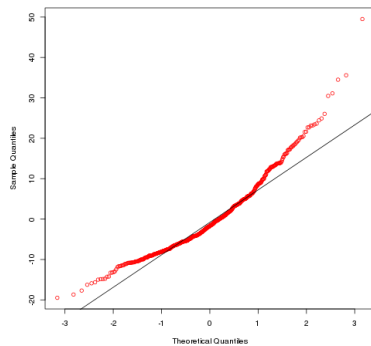
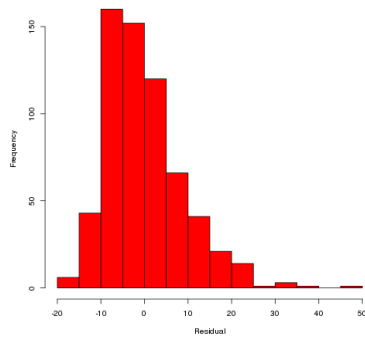
■ ≥ 0.5
■ ≥ 0.3
■ ≥ 0.1
■ ≥ 0.0
 N/A not applicable

Pearson product-moment correlation coefficients (r)

Analysis

Improving correlations

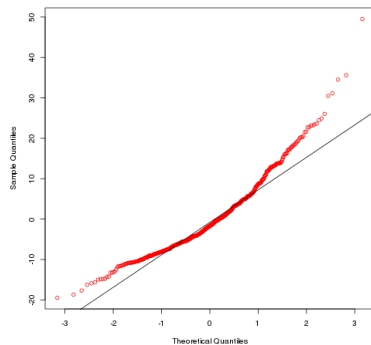
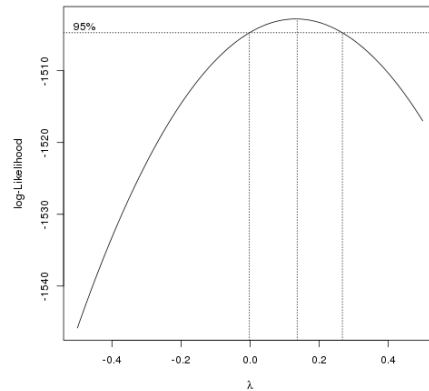
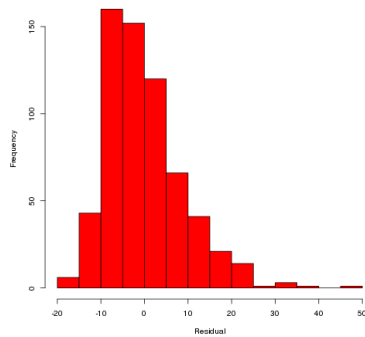
- Histograms and Q-Q plots reveal whether regression residuals follow a normal distribution



Analysis

Improving correlations

- Histograms and Q-Q plots reveal whether regression residuals follow a normal distribution
- Box-Cox method* finds optimal power parameter λ to transform measurements to improve normality



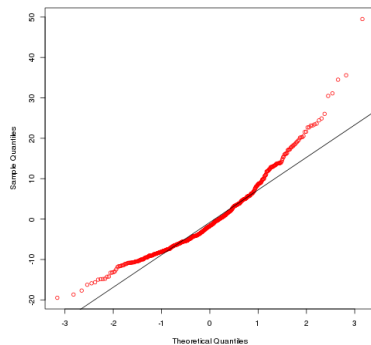
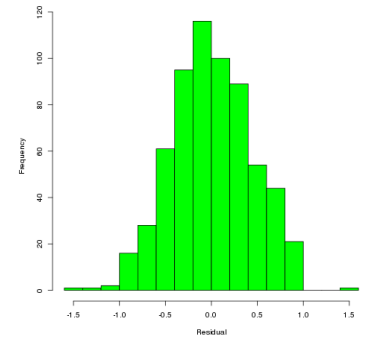
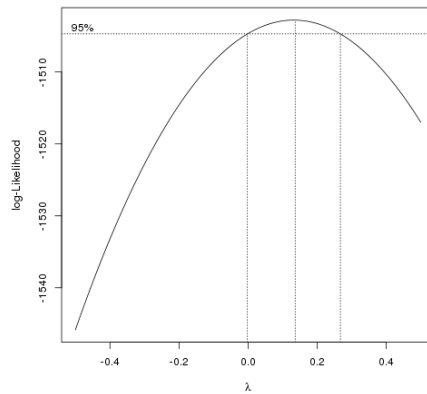
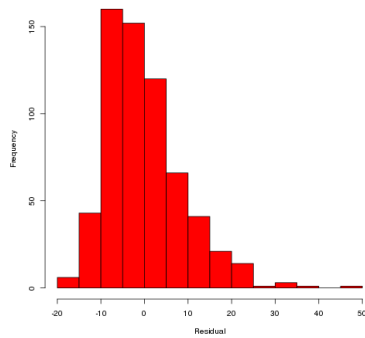
Power transform (λ)

**An Analysis of Transformations*
George E. P. Box, David R. Cox
Journal of the Royal Statistical Society, 1964

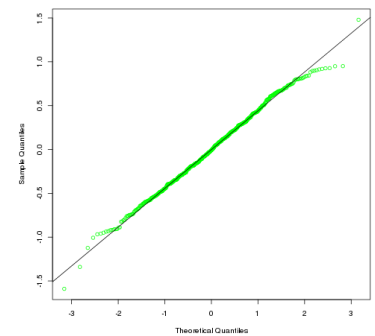
Analysis

Improving correlations

- Histograms and Q-Q plots reveal whether regression residuals follow a normal distribution
- Box-Cox method* finds optimal power parameter λ to transform measurements to improve normality
- For curvature $\lambda=0.25$ and medial axis distance $\lambda=2.0$



Power transform (λ)



**An Analysis of Transformations*
George E. P. Box, David R. Cox
Journal of the Royal Statistical Society, 1964

Overview

Surface perception user study

Analysis of participant study data

Predicting perception error and application

Predicting Error

- *Supervised learning*: We use the study data to create a predictive model for task error
- *Predictors*: independent variables, measurements we can make at surface points (e.g. curvature, local thickness)
- *Response*: dependent variable, the outcome – task error

Predicting Error

- *Linear models* take the following form:

$$f(\mathbf{x}) = \hat{\beta}_0 + \sum_{j=1}^p \hat{\beta}_j h_j(\mathbf{x})$$

p – the number of predictors

\mathbf{x} – a vector of p predictor measurements

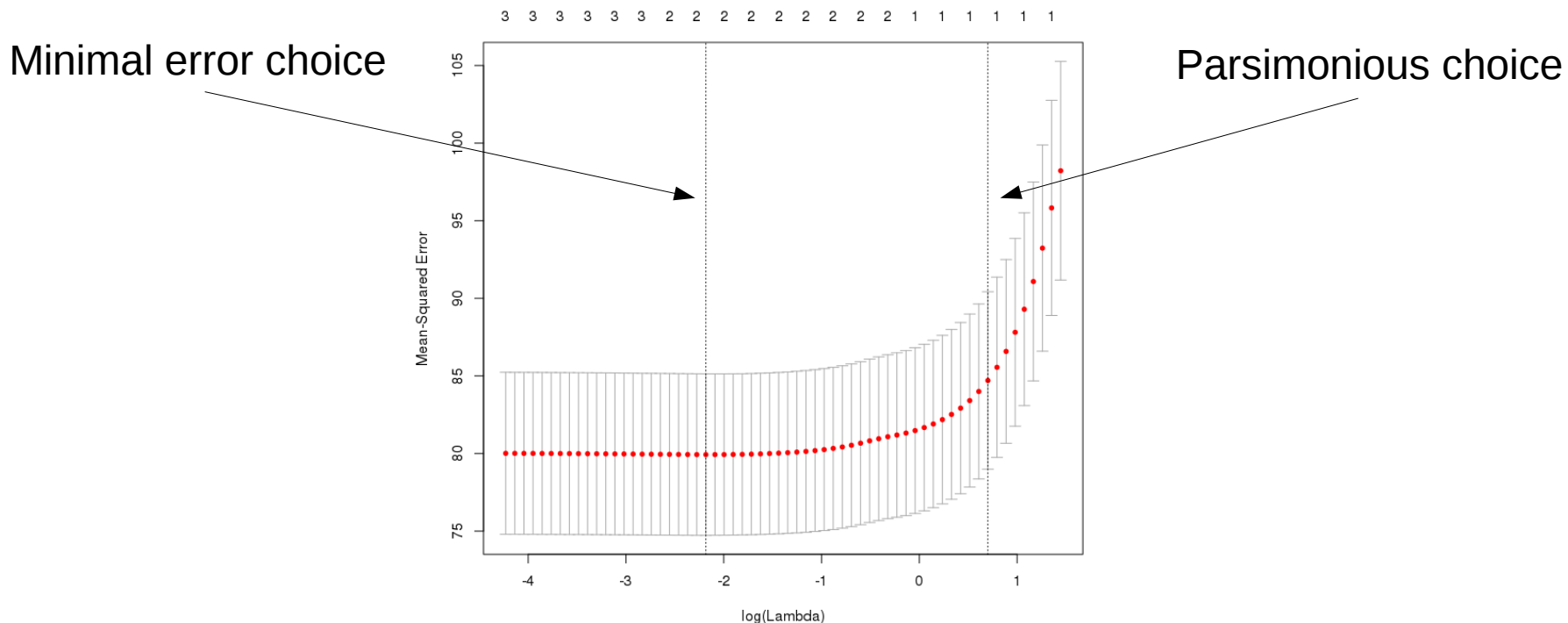
h_j – each predictor j 's transformation function

$\hat{\beta}$ – vector of $(p+1)$ parameters of the linear model

Predicting Error

Regularization

- LASSO* minimizes the L1-norm of $\hat{\beta}$
- Model parameters selected *parsimoniously*: compromise between few predictors *and* low error of fit



**Regression shrinkage and selection via the lasso*
Robert Tibshirani
Journal of the Royal Statistical Society, 1996

Predicting Error

Regularization

- LASSO [Tibshirani 1996] minimizes the L1-norm of $\hat{\beta}$
- Model parameters selected *parsimoniously*: compromise between few predictors *and* low error of fit

Validation

- We perform k -fold cross-validation (for $k=10$)
 - The n input samples divided into 10 equally sized *folds*
 - Linear model trained using samples from 9 folds, last used for testing
 - Repeat 10 times, using each fold for testing once
- *Estimated prediction error* - the mean absolute error over all 10 folds

Predicting Error

Models and Performance

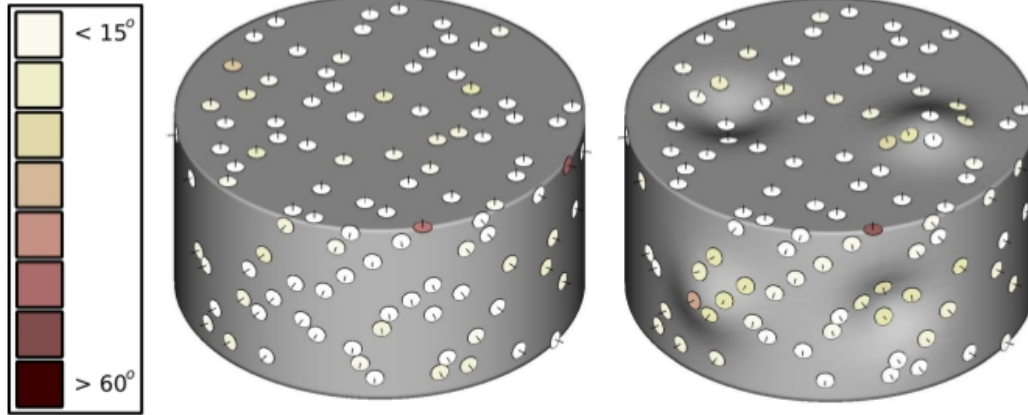
Diagram illustrating the mapping of features to regression coefficients:

- curvature (κ_1, κ_2 , Gaussian, mean) → $\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4$
- medial axis distance → $\hat{\beta}_5$
- local thickness → $\hat{\beta}_6$
- centroid distance → $\hat{\beta}_7, \hat{\beta}_8$
- view-normal angle difference → $\hat{\beta}_9$
- abstraction distance → $\hat{\beta}_{10}$
- abstraction angle difference → $\hat{\beta}_{10}$
- intercept → $\hat{\beta}_0$

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
Fixed surface	13.8	4.69	-	-
MPS	9.07	4.75	40.9	.	0.02	126	0.27
XYZ	16.0	6.79
radial	27.0	-37.3	29.5	14.3	0.12	194	0.15
crdbrd	9.75	.	4.53	.	7.83	.	12.0	.	0.02	209	0.19
Rotated surface	11.9	5.69	.	0.26	-	-
MPS	10.25	3.59	33.3	.	.	111	0.29
XYZ	10.7	7.73	.	0.40	-0.01
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18

Predicting Error

Models and Performance



Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
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crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18

$(\kappa_1, \kappa_2, \text{Gaussian, mean})$
 curvature

Predicting Error

Models and Performance

curvature (κ_1, κ_2 , Gaussian, mean)

medial axis distance

local thickness

centroid distance

view-normal angle difference

abstraction distance

abstraction angle difference

intercept

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
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Predicting Error

Models and Performance

curvature (κ_1, κ_2 , Gaussian, mean)

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abstraction distance

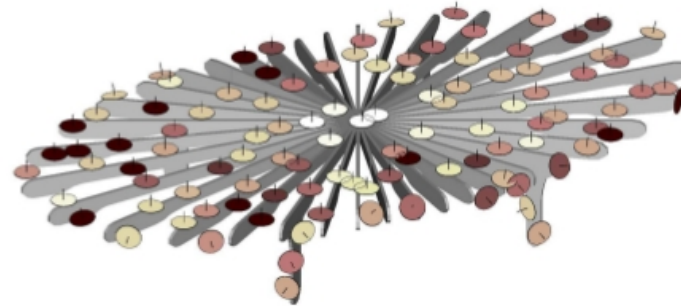
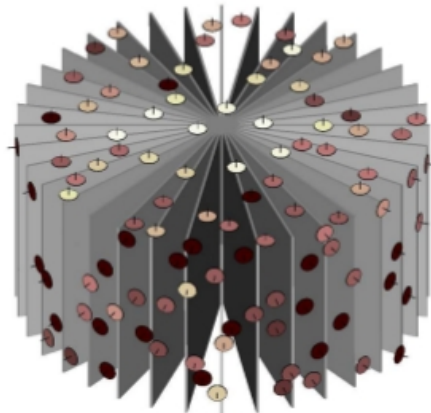
abstraction angle difference

intercept

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
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Predicting Error

Models and Performance



Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
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crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18

centroid distance

Predicting Error

Models and Performance

curvature (κ_1, κ_2 , Gaussian, mean)

medial axis distance

local thickness

centroid distance

view-normal angle difference

abstraction distance

abstraction angle difference

intercept

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
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crdbrd	9.75	.	4.53	.	7.83	.	12.0	.	0.02	209	0.19
Rotated surface	11.9	5.69	.	0.26	-	-
MPS	10.25	3.59	33.3	.	.	111	0.29
XYZ	10.7	7.73	.	0.40	-0.01
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18

Predicting Error

Models and Performance

curvature (κ_1, κ_2 , Gaussian, mean)

medial axis distance

local thickness

centroid distance

view-normal angle difference

abstraction distance

abstraction angle difference

intercept

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$
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MPS	10.25	3.59	33.3	.	.	111	0.29
XYZ	10.7	7.73	.	0.40	-0.01
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18

Predicting Error

Models and Performance

- MAPE – Mean Absolute **Predicted Error**
- MABE – Mean Absolute **Base Error**
- “Improved” - 1.0 – MAPE/MABE (expressed as percentage)

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	MAPE	MABE	Improved
Fixed surface	13.8	4.69	–	–	6.43	7.64	16%
MPS	9.07	4.75	40.9	.	0.02	126	0.27	10.2	19.5	47%
XYZ	16.0	6.79	9.08	11.0	18%
radial	27.0	-37.3	29.5	14.3	0.12	194	0.15	11.6	14.3	19%
crdbrd	9.75	.	4.53	.	7.83	.	12.0	.	0.02	209	0.19	14.1	19.2	27%
Rotated surface	11.9	5.69	.	0.26	–	–	5.81	7.74	25%
MPS	10.25	3.59	33.3	.	.	111	0.29	10.5	19.0	45%
XYZ	10.7	7.73	.	0.40	-0.01	7.36	10.7	32%
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11	9.98	11.3	12%
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18	12.8	16.6	23%

Predicting Error

Models and Performance

- MAPE – Mean Absolute Predicted Error
- MABE – Mean Absolute Base Error
- “Improved” - 1.0 – MAPE/MABE (expressed as percentage)
- Greatest improvement for MPS conditions

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	MAPE	MABE	Improved
Fixed surface	13.8	4.69										6.43	7.64	16%
MPS	9.07	4.75	40.9	.	0.02	126	0.27	10.2	19.5	47%
XYZ	16.0	6.79	9.08	11.0	18%
radial	27.0	-37.3	29.5	14.3	0.12	194	0.15	11.6	14.3	19%
crdbrd	9.75	.	4.53	.	7.83	.	12.0	.	0.02	209	0.19	14.1	19.2	27%
Rotated surface	11.9	5.69	.	0.26	5.81	7.74	25%
MPS	10.25	3.59	33.3	.	.	111	0.29	10.5	19.0	45%
XYZ	10.7	7.73	.	0.40	-0.01	7.36	10.7	32%
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11	9.98	11.3	12%
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18	12.8	16.6	23%

Predicting Error

Models and Performance

- MAPE – Mean Absolute Predicted Error
- MABE – Mean Absolute Base Error
- “Improved” - 1.0 – MAPE/MABE (expressed as percentage)
- Greatest improvement for MPS conditions
- Significant improvements (16%, 25%) for surface conditions

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	MAPE	MABE	Improved
Fixed surface	13.8	4.69	–	–	6.43	7.64	16%
MPS	9.07	4.75	40.9	.	0.02	126	0.27	10.2	19.5	47%
XYZ	16.0	6.79	9.08	11.0	18%
radial	27.0	-37.3	29.5	14.3	0.12	194	0.15	11.6	14.3	19%
crdbrd	9.75	.	4.53	.	7.83	.	12.0	.	0.02	209	0.19	14.1	19.2	27%
Rotated surface	11.9	5.69	.	0.26	–	–	5.81	7.74	25%
MPS	10.25	3.59	33.3	.	.	111	0.29	10.5	19.0	45%
XYZ	10.7	7.73	.	0.40	-0.01	7.36	10.7	32%
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11	9.98	11.3	12%
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18	12.8	16.6	23%

Predicting Error

Models and Performance

- MAPE – Mean Absolute Predicted Error
- MABE – Mean Absolute Base Error
- “Improved” - 1.0 – MAPE/MABE (expressed as percentage)
- Greatest improvement for MPS conditions
- Significant improvements (16%, 25%) for surface conditions
- **Least improvement for radial conditions**

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	MAPE	MABE	Improved
Fixed surface	13.8	4.69	–	–	6.43	7.64	16%
MPS	9.07	4.75	40.9	.	0.02	126	0.27	10.2	19.5	47%
XYZ	16.0	6.79	9.08	11.0	18%
radial	27.0	-37.3	29.5	14.3	0.12	194	0.15	11.6	14.3	19%
crdbrd	9.75	.	4.53	.	7.83	.	12.0	.	0.02	209	0.19	14.1	19.2	27%
Rotated surface	11.9	5.69	.	0.26	–	–	5.81	7.74	25%
MPS	10.25	3.59	33.3	.	.	111	0.29	10.5	19.0	45%
XYZ	10.7	7.73	.	0.40	-0.01	7.36	10.7	32%
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11	9.98	11.3	12%
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18	12.8	16.6	23%

Predicting Error

Improving planar abstractions

- We used the abstraction distance predictor within our MPS predictive models and incorporated it into the MPS abstraction algorithm*

Condition	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	$\hat{\beta}_9$	$\hat{\beta}_{10}$	MAPE	MABE	Improved
Fixed surface	13.8	4.69	-	-	6.43	7.64	16%
MPS	9.07	4.75	40.9	.	0.02	126	0.27	10.2	19.5	47%
XYZ	16.0	6.79	9.08	11.0	18%
radial	27.0	-37.3	29.5	14.3	0.12	194	0.15	11.6	14.3	19%
crdbrd	9.75	.	4.53	.	7.83	.	12.0	.	0.02	209	0.19	14.1	19.2	27%
Rotated surface	11.9	5.69	.	0.26	-	-	5.81	7.74	25%
MPS	10.25	3.59	33.3	.	.	111	0.29	10.5	19.0	45%
XYZ	10.7	7.73	.	0.40	-0.01	7.36	10.7	32%
radial	19.5	1.91	.	.	.	-44.3	10.6	6.36	0.10	177	0.11	9.98	11.3	12%
crdbrd	9.23	7.90	8.08	-3.01	.	-9.44	2.72	.	0.14	139	0.18	12.8	16.6	23%

**Slices: A Shape-proxy Based on Planar Sections*

James McCrae, Karan Singh, Niloy Mitra
SIGGRAPH Asia, 2011

Predicting Error

Improving planar abstractions

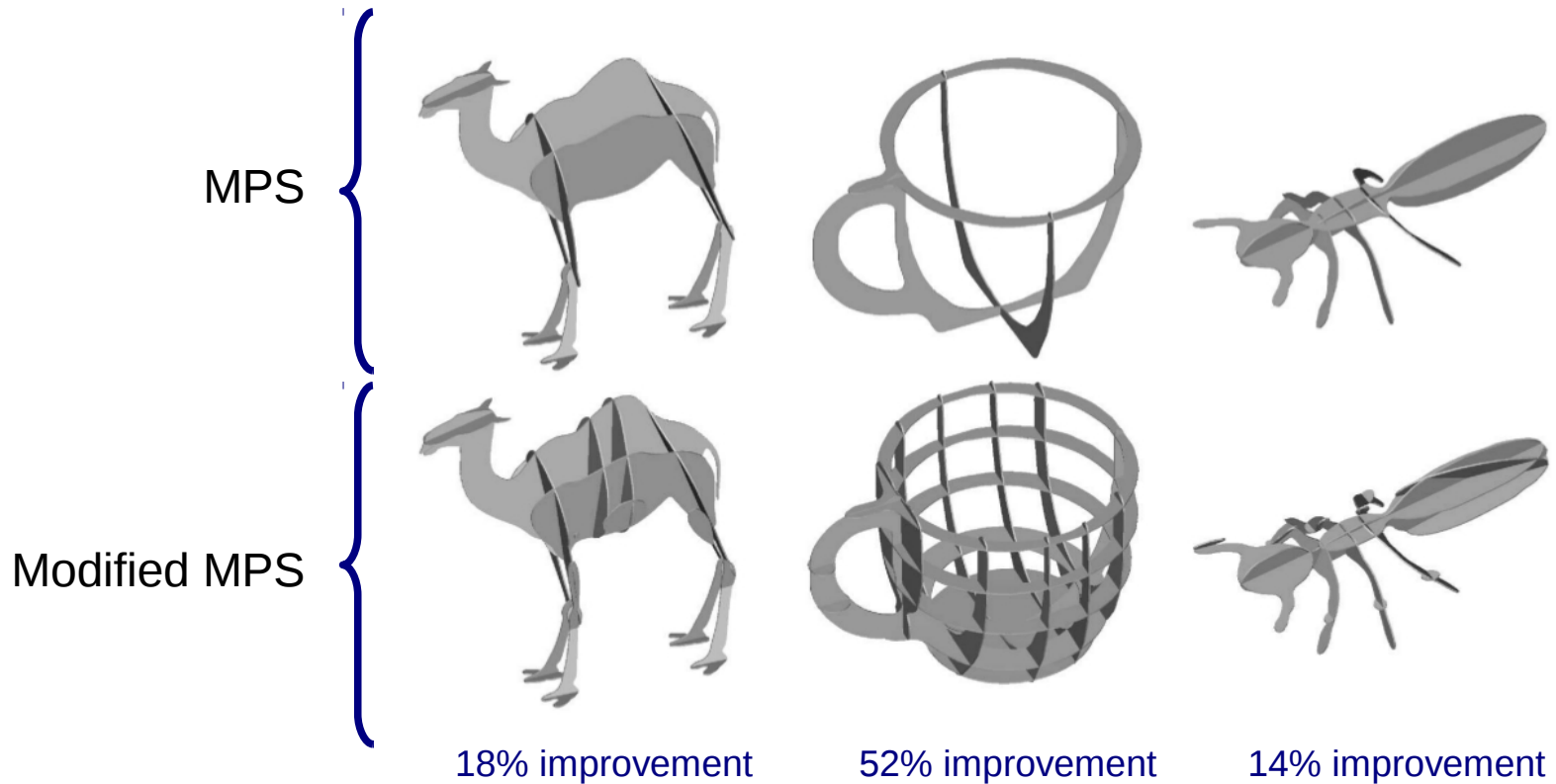
- We used the abstraction distance predictor within our MPS predictive models and incorporated it into the MPS abstraction algorithm*



Predicting Error

Improving planar abstractions

- We used the abstraction distance predictor within our MPS predictive models and incorporated it into the MPS abstraction algorithm*
- Smaller crowd-sourced study revealed notable improvements



Summary

Key Contribution:

An investigation of the visual perception of surfaces represented by planar abstractions

- Design of large crowd-sourced user study
- Identified a variety of geometric sources of error in analysis
- Predictive model parameters learned from study data, models perform significantly better than base estimates
- Demonstrated predictive models can be used to modify existing planar abstraction algorithms in order to improve surface perception

Acknowledgements

Individuals:

- Vangelis Kalogerakis
- W. John Braun
- Kristian Hildebrand
- Dongming Yan
- Daniela Giorgi

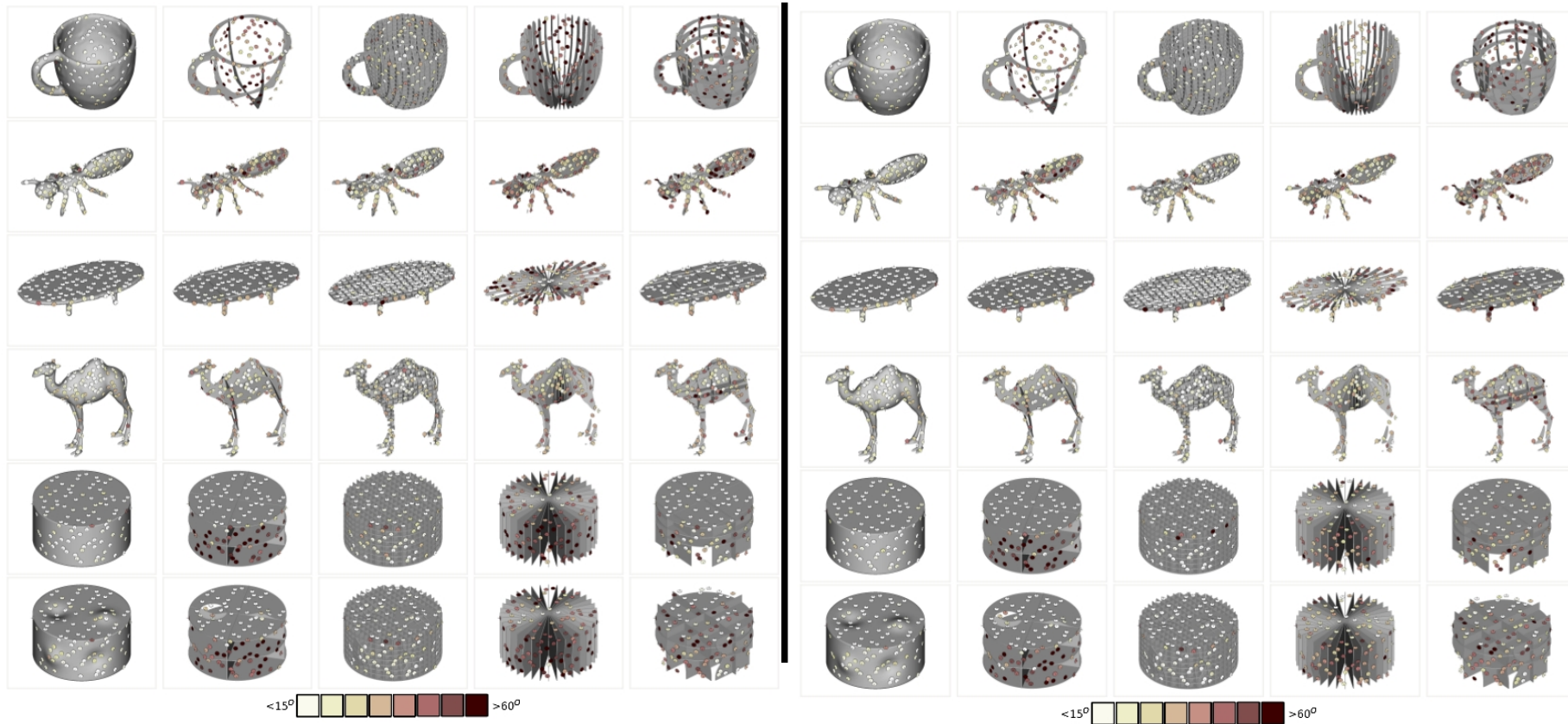
Organizations:

Graphisme, animation et nouveaux médias



Thank You

Questions or comments?



Study data, program code and other resources can be found at:
<http://www.dgp.toronto.edu/~mccrae/>

Extra Slides...

Visual Perception of Shape



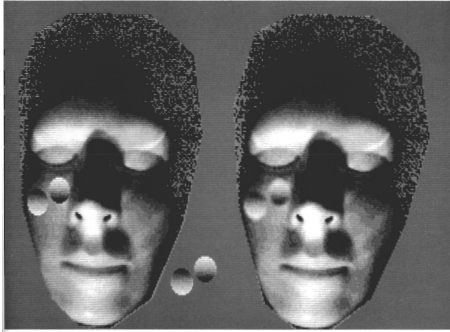
The perception of the visible world

James J. Gibson

Houghton Mifflin, 1950

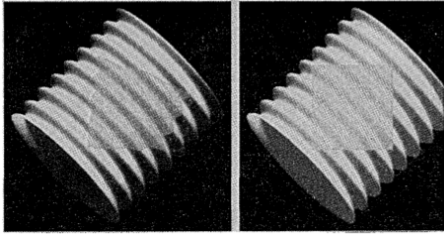
- First formal attempts to study how different types of image structures (lines, gradients, patterns, etc.) inform the human visual system to give perceptual knowledge of 3D shape

Visual Perception of Shape



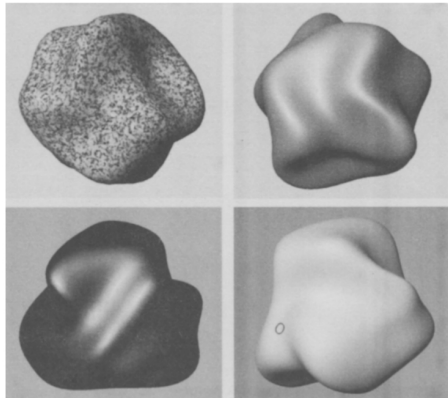
Perceiving shape from shading

Vilayanur S. Ramachandran
Scientific American, 1988



Perception of surface contours and surface shape: from computation to psychophysics

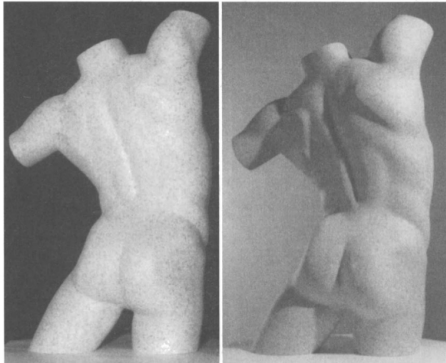
David C. Knill
Journal of the Optical Society of America, 1992



The perception of surface orientation from multiple sources of optical information

J. Farley Norman, James T. Todd, Flip Phillips
Perception and Psychophysics, 1995

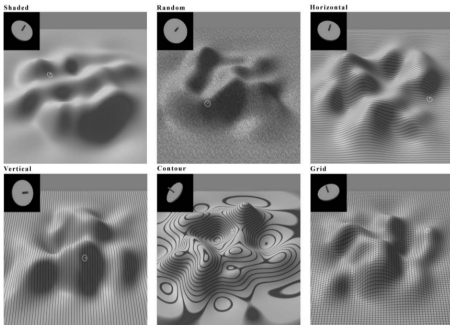
Visual Perception of Shape



Effects of Changing Viewing Conditions on the Perceived Structure of Smoothly Curved Surfaces

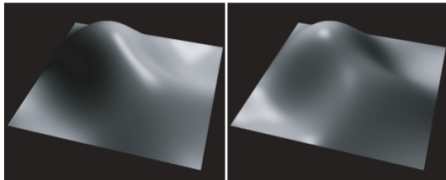
James T. Todd, Jan J. Koenderink, Andrea J. van Doorn, Astrid M. L. Kappers

Journal of Experimental Psychology, 1996



View Direction, Surface Orientation and Texture Orientation for Perception of Surface Shape

Graeme Sweet, Colin Ware
Graphics Interface, 2004

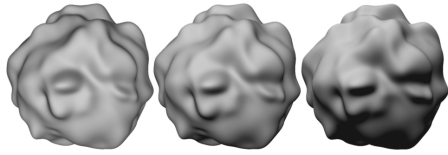


Distortion in 3D shape estimation with changes in illumination

Franck Caniard, Roland W. Fleming

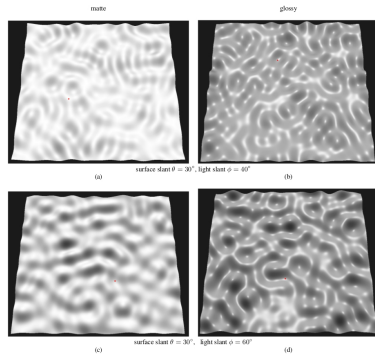
ACM Applied Perception in Graphics and Visualization, 2007

Visual Perception of Shape



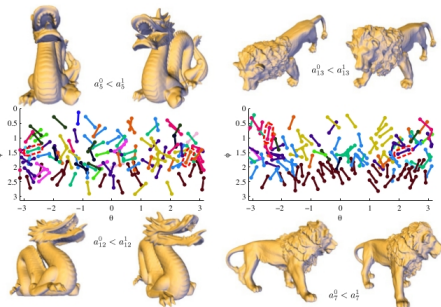
The Assumed Light Direction for Perceiving Shape from Shading

James P. O'Shea, Martin S. Banks, Maneesh Agrawala
ACM Applied Perception in Graphics and Visualization, 2008



How Does Lighting Direction Affect Shape Perception of Glossy and Matte Surfaces?

Arthur Faisman, Michael S. Langer
ACM Symposium on Applied Perception, 2013
(later this session)



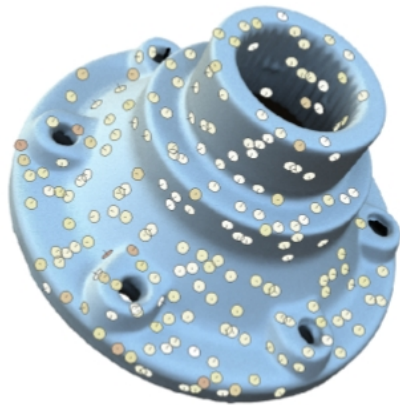
Perceptual models of viewpoint preference

Adrian Secord, Jingwan Lu, Adam Finkelstein,
Manish Singh, Andrew Nealen
ACM Transactions on Graphics, 2011

Visual Perception of Shape

How well do line drawings depict shape?

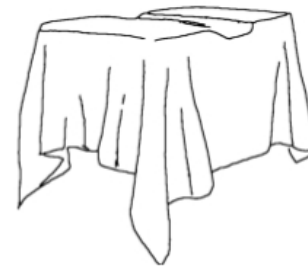
Forrester Cole, Kevin Sanik, Doug DeCarlo, Adam Finkelstein,
Thomas Funkhouser, Szymon Rusinkiewicz, Manish Singh
ACM Transactions on Graphics (Proc. SIGGRAPH), 2009



ridges and valleys



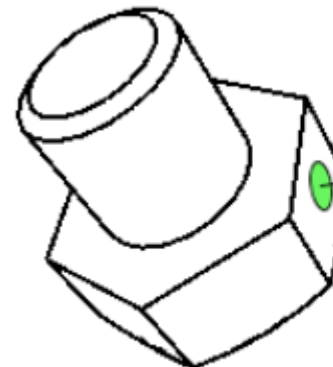
suggestive contours



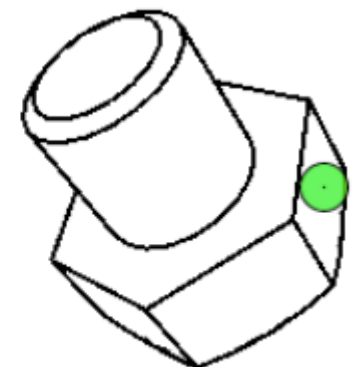
artist's drawing



shaded



good



bad

Overview

User Study

- Candidate planar abstractions
- Methodology
- Data collection
- Data verification

Analysis of Results

- Initial analysis
- On the bas-relief ambiguity
- Participant-specific correlations
- Surface-specific correlations
- Abstraction-specific correlations
- Improving correlations

Predicting Error

- Linear models
- Regularization
- Validation
- Performance of linear models
- Improving planar abstractions

User Study

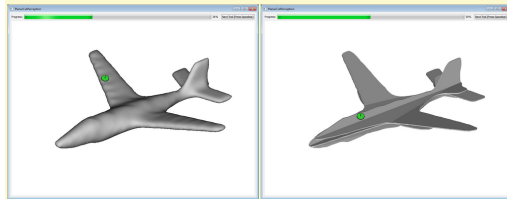
Methodology

- Participants on Amazon's “Mechanical Turk” view a webpage with basic instructions, broad description of the task, and software download link

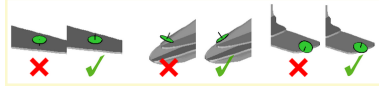
PlanarCutPerception - An academic study on how well one can perceive the surface of a 3D object represented by an abstract model

NOTE: If you participated in the first run of the “PlanarCutPerception” study, you are not eligible to participate in this run, sorry! (The application will display a warning and exit if you provide a Worker ID from the first study.)

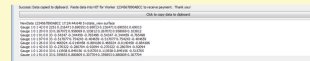
We would like to study how well one can perceive the surface of a 3D object represented by an abstract model. Each run of the study should take about 10 minutes to complete, and less time with a bit of practice. A Windows-based computer is required to run the application. Your 12 to 14-character “Worker ID” (included in every HIT), is required when naming the study - you will need to be able to provide this. Participants are encouraged to run through the study up to a maximum of 10 times!



You will either be shown the true surface of the object, or an “abstraction” - a set of planar (or “flat and straight”) pieces that are meant to abstractly represent the surface - like this example of an airplane. If you are shown the surface, we want you to rotate the green gauges (a disc with a tip coming out) such that the disc lies flat on the surface, and the tip points directly away from the surface. If you are shown the “abstraction”, we want you to imagine the hidden surface and rotate the gauge to the disc lies on the imagined surface at that point.



- After the 60 gauges have been set, their precision will be evaluated (not being too sloppy or careless). If you fail to meet the conditions, a message will prompt you to re-run the study and try again. Otherwise, a message will indicate your success and the study data to submit will be provided (and automatically copied to your clipboard for pasting here.)



- Right-click and paste the study data into the text box at the bottom of this HIT and press “Submit” button
- You are encouraged to repeat the study up to 10 times - run the application again and create new data to submit in another HIT!

Notes:

- To be **approved**, the Worker ID entered into the study application must match the Worker ID when the HIT is submitted. (Mechanical Turk Worker IDs have 12 to 14 alpha-numeric characters)
- To be **approved**, you must correctly **copy/paste** the generated data from the study program in your HIT submission (if the text contains many lines that start with “Gauge”, etc., then you probably copied the data in correctly)
- To be **approved**, submitted data for each HIT must come from a unique run of the study (do not copy data from one run of the study into multiple HITs as the copies will be **rejected**)
- To be **approved**, no more than 10 HITs from a specific Worker ID will be accepted - submissions beyond 10 from a particular Worker ID will be **rejected**. (After 10 successful runs of the study, the study application will not allow you to run the study any more to prevent this from happening accidentally)
- The data you submit includes only information about your publically-known Worker ID, the time you ran the study, and various statistics on the settings for the 60 gauges. No personal information is collected.

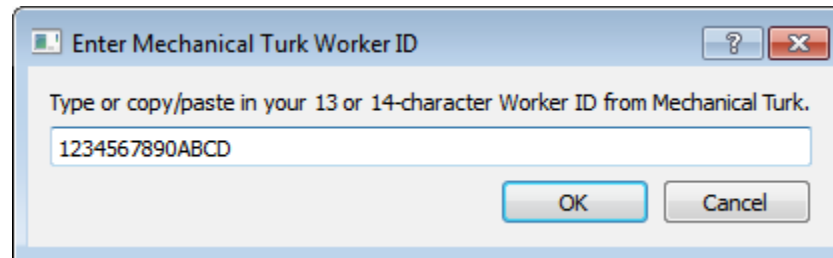
Copy/paste study data here:

We thank you for your participation in this university study! (HIT #5{participant})

User Study

Methodology

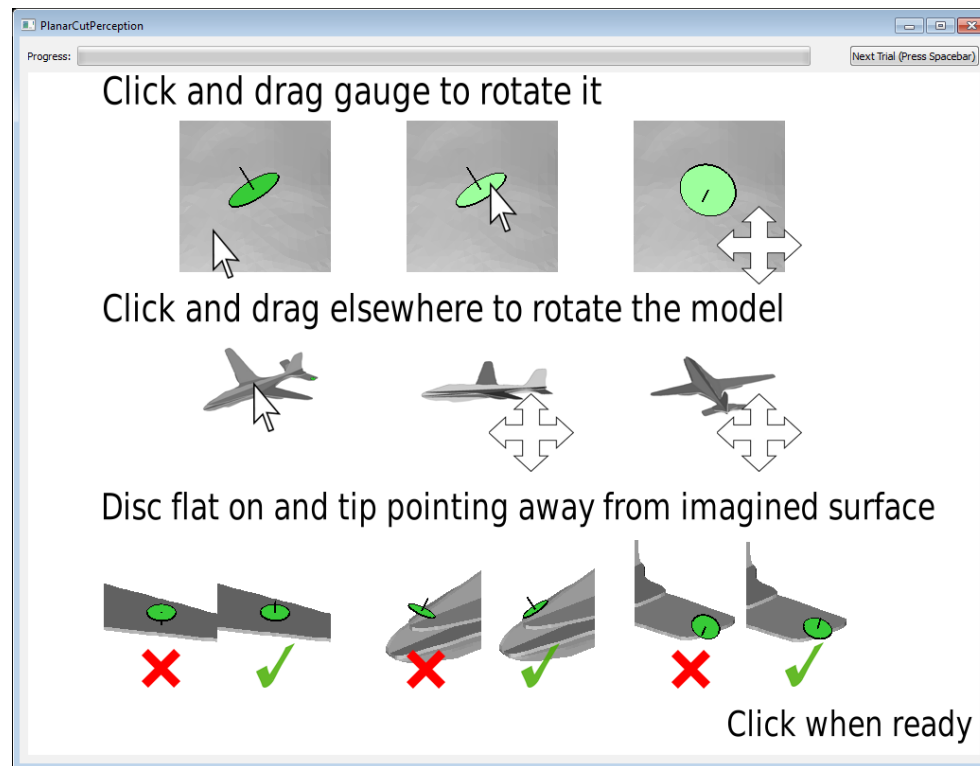
- Participants on Amazon's "Mechanical Turk" view a webpage with basic instructions, a description of the task, and a software download link
- Participants first enter their unique **worker ID**, which is used to define conditions (task and representation) for the participant



User Study

Methodology

- Participants on Amazon's "Mechanical Turk" view a webpage with basic instructions, a description of the task, and a software download link
- Participants first enter their unique *worker ID*, which is used to define conditions (task and representation) for the participant
- **Initial instructions describe task and provide examples**



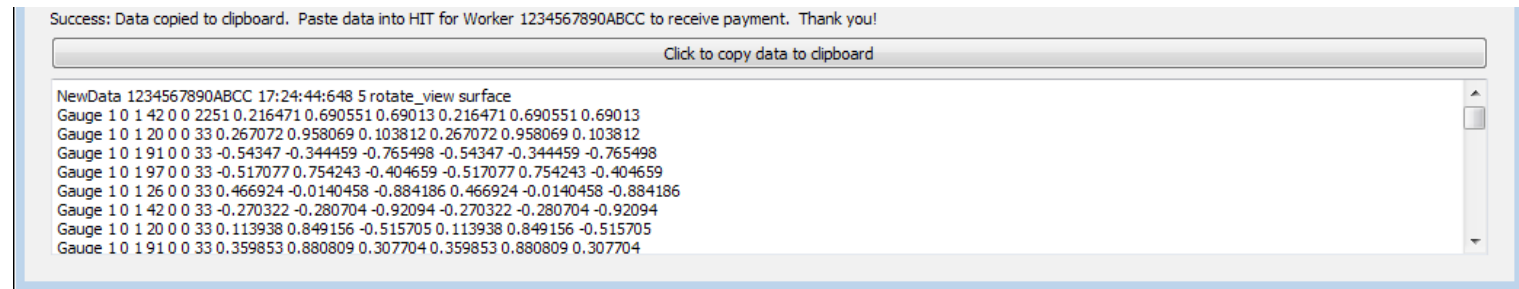
User Study

Methodology

- Participants on Amazon's "Mechanical Turk" view a webpage with basic instructions, a description of the task, and a software download link
- Participants first enter their unique **worker ID**, which is used to define conditions (task and representation) for the participant
- Initial instructions describe task and provide examples
- Participants set 60 gauges (30 pairs) in total

Data Collection

- Data generated from study application is submitted by participant to Mechanical Turk



Success: Data copied to clipboard. Paste data into HIT for Worker 1234567890ABCC to receive payment. Thank you!

[Click to copy data to clipboard](#)

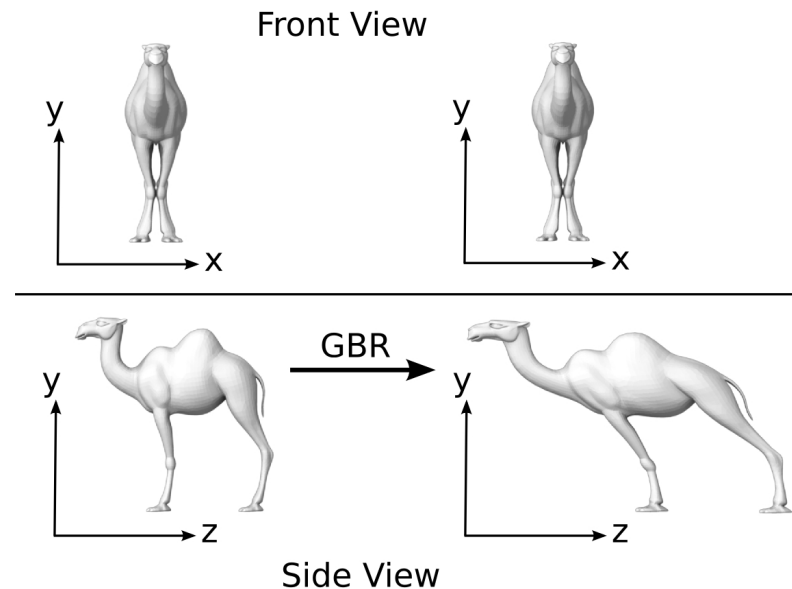
```
NewData 1234567890ABCC 17:24:44:648 5 rotate_view surface
Gauge 1 0 1 42 0 0 2251 0.216471 0.690551 0.69013 0.216471 0.690551 0.69013
Gauge 1 0 1 20 0 0 33 0.267072 0.958069 0.103812 0.267072 0.958069 0.103812
Gauge 1 0 1 91 0 0 33 -0.54347 -0.344459 -0.765498 -0.54347 -0.344459 -0.765498
Gauge 1 0 1 97 0 0 33 -0.517077 0.754243 -0.404659 -0.517077 0.754243 -0.404659
Gauge 1 0 1 26 0 0 33 0.466924 -0.0140458 -0.884186 0.466924 -0.0140458 -0.884186
Gauge 1 0 1 42 0 0 33 -0.270322 -0.280704 -0.92094 -0.270322 -0.280704 -0.92094
Gauge 1 0 1 20 0 0 33 0.113938 0.849156 -0.515705 0.113938 0.849156 -0.515705
Gauge 1 0 1 91 0 0 33 0.359853 0.880809 0.307704 0.359853 0.880809 0.307704
```

Analysis of Results

Initial Analysis

- Outliers (removal if mean error with group > 3 standard deviations)
 - One participant with mean error > 120 degrees
 - 4 of 182 participants in total classified as outliers and removed
- Average error
- Average error (for flat models/regions only)
- **On the bas-relief ambiguity**

$$\mathbf{p} = (x, y, f(x, y)) \xrightarrow{\lambda, \mu, \nu} \bar{\mathbf{p}} = (x, y, \lambda f(x, y) + \mu x + \nu y)$$



“Generalized Bas-Relief” transform for $\lambda = 1.25$, $\mu = 0$, $\nu = -0.5$

Analysis of Results

Initial Analysis

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- Average error (for flat models/regions only)
- **On the bas-relief ambiguity**

$$\mathbf{n} \xrightarrow{\lambda, \mu, \nu} \bar{\mathbf{n}}$$

$$\frac{1}{\lambda} \begin{bmatrix} 1 & 0 & -\mu \\ 0 & 1 & -\nu \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \bar{n}_x \\ \bar{n}_y \\ \bar{n}_z \end{bmatrix}$$

Normal directions are also transformed

Analysis of Results

Initial Analysis

- Outliers (removal if mean error with group > 3 standard deviations)
 - One participant with mean error > 120 degrees
 - 4 of 182 participants in total classified as outliers and removed
- Average error
- Average error (for flat models/regions only)
- On the bas-relief ambiguity
 - Rotated view task - does not require GBR, no depth ambiguity
 - Fixed view task - performance comparable to rotated view task
 - Optimal GBR parameters can reduce error > 5 degrees artificially, we opted not to

Average	20.2	36.2	24.3	52.8	38.0
	20.1	34.4	21.0	39.9	41.0

Analysis of Results

General correlations

- Gauge consistency (participant agreement)

Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations
Fixed surface	0.95	0.17	0.51		N/A
MPS		0.31	0.51		N/A
XYZ	0.47	0.21	0.59		N/A
radial		0.25		0.15	N/A
crdbrd	0.52	0.33	0.49		N/A
Rotated surface	0.57	0.25	0.59		
MPS		0.31	0.50	0.17	
XYZ	0.63	0.22	0.57		
radial		0.18	0.45		
crdbrd		0.15	0.42		

■ ≥ 0.5 ■ ≥ 0.3 ■ ≥ 0.1 ■ ≥ 0.0 N/A not applicable

Pearson product-moment correlation coefficients (r)

Analysis of Results

Surface-specific correlations

- Curvature (κ_1 , κ_2 , Gaussian, mean)
- Local thickness
- Medial axis distance
- Centroid distance
- View-normal angle difference

Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations	absolute κ_1	absolute κ_2	Gaussian curvature	mean curvature	local thickness	medial axis distance	centroid distance	view-norm angle difference
Fixed surface	0.95	0.17	0.51		N/A	0.51	0.42	0.44	0.49		-0.13		-0.17
MPS		0.31	0.51		N/A	0.42	0.26	0.19	0.42	0.49	-0.20	0.23	
XYZ	0.47	0.21	0.59		N/A	0.54	0.42	0.46	0.53		-0.24	0.11	-0.14
radial		0.25		0.15	N/A		-0.14	-0.16		0.39	-0.10	0.28	0.21
crdbrd	0.52	0.33	0.49		N/A	0.46	0.41	0.33	0.46	0.14	-0.16	0.15	
Rotated surface	0.57	0.25	0.59			0.60	0.50	0.53	0.57		-0.21		-0.19
MPS		0.31	0.50	0.17		0.40	0.25	0.18	0.39	0.46	-0.20	0.21	-0.13
XYZ	0.63	0.22	0.57			0.64	0.50	0.56	0.62		-0.28		-0.22
radial		0.18	0.45			0.12			0.11	0.22	-0.16	0.21	0.15
crdbrd		0.15	0.42			0.43	0.37	0.31	0.42		-0.20	0.10	

■ ≥ 0.5
■ ≥ 0.3
■ ≥ 0.1
■ ≥ 0.0
 N/A not applicable

Pearson product-moment correlation coefficients (r)

Analysis of Results

Abstraction-specific correlations

- Abstraction distance
- Abstraction angle difference

Condition	user persistence	gauge persistence	gauge consistency	trial duration	number of view rotations	absolute κ_1	absolute κ_2	Gaussian curvature	mean curvature	local thickness	medial axis distance	centroid distance	view-norm angle difference	abstraction distance	abstraction angle difference
Fixed surface	0.95	0.17	0.51		N/A	0.51	0.42	0.44	0.49		-0.13		-0.17	N/A	N/A
MPS		0.31	0.51		N/A	0.42	0.26	0.19	0.42	0.49	-0.20	0.23		0.39	0.72
XYZ	0.47	0.21	0.59		N/A	0.54	0.42	0.46	0.53		-0.24	0.11	-0.14	0.13	-0.26
radial		0.25		0.15	N/A		-0.14	-0.16		0.39	-0.10	0.28	0.21	0.20	0.30
crdbrd	0.52	0.33	0.49		N/A	0.46	0.41	0.33	0.46	0.14	-0.16	0.15		0.17	0.43
Rotated surface	0.57	0.25	0.59			0.60	0.50	0.53	0.57		-0.21		-0.19	N/A	N/A
MPS		0.31	0.50	0.17		0.40	0.25	0.18	0.39	0.46	-0.20	0.21	-0.13	0.37	0.72
XYZ	0.63	0.22	0.57			0.64	0.50	0.56	0.62		-0.28		-0.22		-0.35
radial		0.18	0.45			0.12			0.11	0.22	-0.16	0.21	0.15	0.19	0.17
crdbrd		0.15	0.42			0.43	0.37	0.31	0.42		-0.20	0.10			0.43

■ ≥ 0.5
■ ≥ 0.3
■ ≥ 0.1
■ ≥ 0.0
 N/A not applicable

Pearson product-moment correlation coefficients (r)