Geometric Modeling in Shape Space

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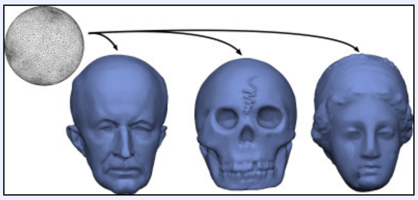
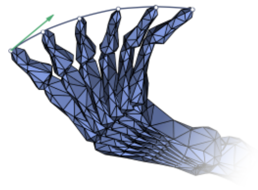
A discrete model of shape space



We model geometric objects – shapes – by mapping a fixed connectivity (a simplicial complex) to each object under consideration. The vertex correspondence problem is solved in an independent preprocessing step.

Deformations – curves in shape space

During deformation of a shape each vertex travels along a smooth curve. A curve in shape space – a deformation – is a smooth family of such vertex paths.



Differentiating each curve at a common instant of time gives a collection of vectors attached to a shape. Such vector fields are tangent vectors to shape space.

Shapes and metrics

In order to solve geometic problems in shape space one needs algorithms to handle basic constructions of Riemannian geometry like the computation of geodesics, the exponential map, and parallel transport.

Straight lines, i.e., geodesics should correspond to pleasing deformations of a shape. While this may be hard to capture mathematically it shows that the preservation ofi ntrinsic and extrinsic characteristics of the surface of a shape during deformation is a good approximation to this notion.

The goal is to define Riemannian metrics whose geodesic correspond to isometric, rigid, or volume preserving mappings, respectively. Of course this goal can only be reached exactly if the input shapes are isometric. Otherwise we try to achieve this goal in an as-good-as-possible way.

Isometric deformations

In our discrete model a deformation is isometric if the length of each edge is preserved during deformation. This is equivalent to the following property of tangent vectors:

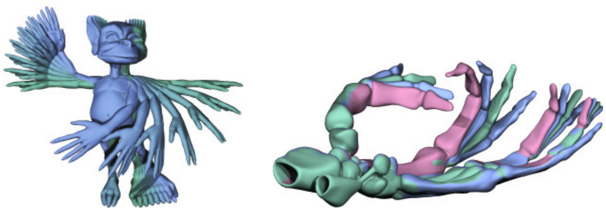
$$\langle X_p - X_q, p - q\rangle = 0, \qquad (p,q) \in E.$$

The symmetric bilinear form

$$\langle\langle X, Y\rangle\rangle := \sum_{(p,q)\in E} \langle X_p - X_q, p - q\rangle\langle Y_p - Y_q, p - q\rangle$$

gives rise to a Riemannian metric whose geodesics correspond to isometric deformations.

Geodesic interpolation and extrapolation



Left: Geodesic interpolation using the as-isometric-as-possible shape metric. Right: Geodesic interpolation with subsequent extrapolation (purple).

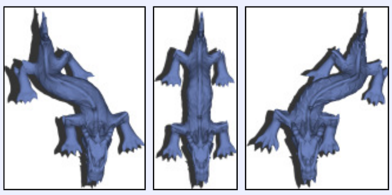
Constrained deformation

Instead of prescribing locations for all vertices of a shape we can fix only a certain subset. Minimizing the energy

$$E = \sum \langle\langle X_i, X_i\rangle\rangle_{M_i} + \langle\langle X_i, X_i\rangle\rangle_{M_{i+1}}$$

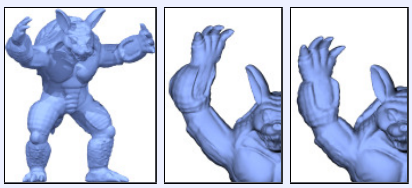
of a curve for this more general kind of boundary value problem results in a natural placement off ree vertices.

More constrained deformations

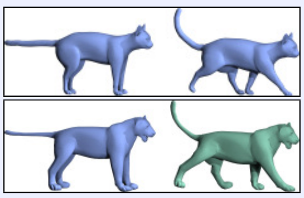


Geodesic interpolation can be used to cope with notions such as symmetry. Symmetric poses correspond to geodesic midpoints.

Fixing vertices in undistorted areas, constrained deformation can reduce overall distortion introduced during modeling.



Deformation transfer via parallel transport



We use the concept of parallel transport to attach a given deformation at a new starting point.

Blue: Input poses. Green: Result of deformation transfer.

Bibliography

[1] Martin Kilian, N. J. Mitra, and H. Pottmann, Geometric modeling in shape space, ACM Trans. Graphics (Proc. SIGGRAPH' 07), vol. 26, no. 3, 2007.

FWF Der Wissenschaftsfonds.

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